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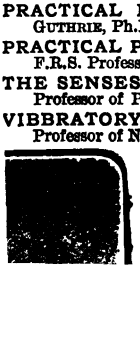

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KEY

TO

THOMSON'S

TREATISE ON

ARITHMETIC

LONDON : PRINTED BY
SPOTTISWOODE AND CO., NEW-STREET SQUARE
AND PARLIAMENT STREET

KEY

TO

THOMSON'S

TREATISE ON

ARITHMETIC

IN THEORY AND PRACTICE

ADAPTED TO THE SEVENTY-SECOND EDITION

AND CONTAINING

Useful Hints for Teachers

EDITED BY HIS SONS

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LONDON

LONGMANS, GREEN, AND CO.

1881

EDITOR'S PREFACE

TO EDITION OF 1881.

In this edition of the Key, alterations and amendments have been made, for the purpose of adapting it to the seventy-second edition of the Arithmetic published in 1880.

University, Glasgow:
April 21, 1881.

AUTHOR'S PREFACE

TO EDITION OF 1848.

THE following pages exhibit the method of working all the exercises and questions in the Author's Treatise on Arithmetic, except a few of the simplest kind. The publication being intended exclusively for the use of teachers, or of persons studying arithmetic without the help of a master, the method of performing each operation is shown, and the results of the principal steps in the various processes are given. In many instances, also, where it appeared more necessary, the work has been given at full length; and, in other instances, different methods of performing the same question are pointed out, and the results of the principal steps exhibited. By these means, the teacher, it is hoped, will derive much advantage from the use of the Key in the daily business of his school; as, by referring to it, he can check the work of the pupil, or detect his mistakes, without the tedious and fatiguing labour of examining each operation in detail; and can thus save much time, which may be profitably employed in the other duties of his situation.

Viewed in this light, the public and open use of a Key, in which the entire work is seldom given, can never be justly regarded as discreditable to the well-qualified teacher. In many of the most respectable schools, particularly in England, it has long been the practice for the arithmetical master to have such a work on his desk, and to use it publicly on every occasion; and, from the increased respectability of teachers generally, it is to be hoped, that, in most instances, with due explanation, no discredit would be attached to the use of such aid. On this point, however, each teacher will use his own discretion, according to the circumstances in which he may be placed.

In this edition of the Key, a few pages are introduced at the end, containing 'Hints for Teachers,' chiefly regarding the formation of new exercises, of such a kind, that the work of them by the pupil may be easily and quickly examined. The Author can state from experience, that much of what is here given, and more especially that which regards simple division, will be found to be of great use to teachers, both in saving time, and in lightening labour.

GLASGOW COLLEGE: *May* 1848.

CONTENTS.

KEY

TO

THOMSON'S ARITHMETIC.

SIMPLE MULTIPLICATION.

(Arithmetic, page 30.)

THE following solutions, except that of exercise 58, contain the several PARTIAL PRODUCTS that arise in the work of each exercise. By means of these, the teacher may ascertain whether the pupil be right in his work, or where he may have committed an error, by causing him to read over the partial products which he has obtained.

Exer.

17. 3832, 2874
18. 43188, 7198, 14396
19. 157080, 219912, 31416
20. 79758, 70896, 8862
21. 42426, 35355, same
22. 465930, same, 372744, same
23. 36837, 28651, same
24. 74070, 98760, 74070
25. 278886, 139443, 418329
26. 133872, 117138
27. 37875, 53025, 37875, 53205
28. 665672, 166418, 249627, 83209
29. 350, 525, 1225
30. 62428, 140463, 46821
31. 59377224, 44532918, 29688612
32. 37057588, 74115176, 46321985, 83379573
33. 32811233, 37498552, 42185871, 4687319
34. 1428371, 204053, 1224318, 204053
35. 950734, 6655138, same

Exer.

36. 279636861, 239688738
37. 664967214, 517196722, 443311476
38. 463219744, 289512340
39. 463219744, 289512340
40. 463219744, 289512340
41. 112334688, 62408160, 12481632
42. 474408150, 632544200, 237204075, 79068025
43. 278846367, 371795156, 650641523
44. 235054872, 293818590, 411346026, 352582308
45. 292336652, 584673304, 365420815, 511589141
46. 576, same, 144; then 82944, same, 20736
47. 3851, 19255, 30808, 11553; then 14830201, 74151005, 118641608, 44490603
48. 395472255, 711850059, 316377804, 474566706, 553661157
49. 63638440, 15909610, 55683635, 71593245
50. 336047124, 392054978, 504070686, 336047124, 448062832
51. 4800456, 5600532, 6400608
52. 3547329540, 4256795448, 2128397724, 5675727264
53. 6142576048, 4387554320, 2632532592, 4387554320, 3510043456, 4387554320
56. 156 strokes in 24 hours; then 156 × 365; 780, 936, 468
57. 365 × 24; 1460, 730; total 8760 hours. 8760 × 60=525600 minutes. 525600 × 240, 21024000, 1051200, total 126144000 ticks per year. 126144000 × 3 = 378432000 ticks in 3 years
58. 774 × 39 = 30186, the number of pages. Then 30186 × 2 × 67=4044924, the number of lines. Hence 4044924 × 10 = 40449240, the number of words: and 4044924 × 47 = 190111428, the number of letters
59. 69342, 138684, 208026, 69342

SIMPLE DIVISION.

(Arithmetic, page 44.)

In the following solutions, all the REMAINDERS are exhibited, except the *last* in each, which is given in the Arithmetic. Hence, if the pupil be required to read the

several remainders that he has obtained in his work, the teacher will discover whether the process is correct, or where it is erroneous.

Exer.

10. 4, 1, 15
11. 24, 23, 12
12. 5, 16, 37
13. 23, 22, 16
14. 30, 56, 60
15. 51, 72, 58
16. 70, 20, 30
17. 16, 64, 64
18. 5, 12, 35, 30
19. 6, 4, 15, 6
20. 8, 84, 30, 1, 13
21. 319, 337, 110, 290, 48, 80
22. 171, 94, 564, 793, 650
23. 833, 168, 782, 566, 221
24. 253, 131, 108, 1084, 10, 105
25. 596, 426, 943, 575, 223, 20, 208
26. 550, 916, 910, 852, 268, 849, 242, 587
27. 589, 47, 471, 330, 378, 129, 562, 509
28. 146, 178, 497, 466, 158, 297, 398, 122
29. 565, 569, 603, 220, 19, 190, 450, 141
30. 182, 308, 48, 489, 337, 340, 364, 604, 727
31. 684, 130, 465, 462, 430, 108, 244, 766, 113
32. 1190, 540, 357, 1050, 399, 204, 781, 239
33. 816, 198, 849, 529, 743, 607, 385, 441
34. 68, 681, 744, 613, 62, 627, 103, 513
35. 257, 9, 91, 57, 571, 587, 747, 639, 413
36. 506, 5061, 6308, 4011, 3194, 2409, 1944, 4679
37. 7099, 2252, 5337, 1821, 1028, 1689
38. 957, 1211, 959, 1227, 1121, 62, 622
39. 1539, 4795, 262, 2623, 5037, 2682
40. 1134, 3581, 4752, 929, 1527, 7507, 5178
41. 983, 943, 545, 1006, 1178, 2892, 2259, 371
42. 3904, 1345, 5915, 6379, 3478, 4632, 1090, 3362
43. 530, 5306, 5573, 1465, 1085, 4069, 6774, 6691, 5859
44. 3300, 4846, 1528, 5901, 2692, 8153, 6442, 8101
45. 211, 335, 684, 612, 783, 712, 887, 755, 426, 703, 800
46. 521, 452, 564, 89, 100, 211, 529, 535, 599, 445, 489

Exer.

47. 279, 27, 270, 622, 683, 594, 405, 589, 351, 46, 461
48. 486, 125, 64, 48, 487, 127, 93, 341, 450, 350, 536
49. 89, 400, 65, 157, 92, 434, 397, 35, 354, 94, 448, 44
50. 189, 321, 75, 357, 34, 347, 327, 135, 175, 183, 259, 233
51. 289, 256, 225, 199, 233, 286, 224, 198, 226, 214, 90, 22
52. 3, 33, 146, 109, 126, 109, 126, 111, 149, 144, 90, 129
53. 435, 6, 63, 150, 54, 60, 120, 237, 441, 66, 180, 354
54. 429, 421, 341, 25, 253, 113, 165, 201, 77, 289, 473, 377
55. 19, 28, 37, 46, 55, 64, 73, 1, 10, 19, 28, 37, 46
56. 271, 523, 127, 541, 307, 154, 82, 91, 181, 352, 604, 208
57. 1, 10, 100, 1, 10, 100, 1, 10, 100, 1, 10, 100
58. 1, 10, 100, 1000, 1, 10, 100, 1000, 1, 10, 100
59. 1, 10, 100, 1000, 10000, 1, 10, 100, 1000, 10000
60. 61731, 35, 355, 3555, 35555, 108643
61. 320987, 592591, 37026, 370265, 431050
62. 32656, 79648, 55749, 63666, 19386
63. 253749, 574533, 510762, 527378, 39212, 392124
64. 325641, 181494, 277487, 84319, 74468, 360317
65. 4, 5, 16, 14, 0, 9, 9, 14
66. 2, 28, 281, 262, 74, 13, 138
67. 50, 501, 637, 247, 721, 210, 352
68. 721, 512, 342, 558, 798
69. 801, 118, 310, 478, 398, 480
70. 131, 94, 125, 33, 67
71. 19, 24, 67, 68, 80, 20
73. 1, 10, 1, 10

FRACTIONS:—INTRODUCTORY CHAPTER.

(Arithmetic, page 55.)

Exer.

29. Remainders, 58, 12, 1, 15
30. ..... 16, 29, 14, 8
34. ..... 16, 14, 7, 8

REDUCTION.

In most of the following solutions, the results in the full work, which are intermediate between the answers and the numbers given in the exercises, are exhibited; so

that the teacher will be able to check the work of the pupil, by causing him to read the several results which he has obtained. The few solutions which proceed differently will be easily understood.

MONEY.

(Arithmetic, page 65.)

Exer.

1. 6820*s.*
2. 1957*s.* 23487*d.*
3. 15662*s.* 187947*d.*
4. 9620*s.*
5. 2777*s.*
7. 2275*s.*
9. 191*s.*
11. 9279*s.* 111355*d.*
12. 34*s.* 409*d.*
13. 47641*d.*
14. 774*s.*
15. 2666*s.* 32000*d.*

Exer.

16. 3965*s.*
17. 308641*d.* 25720
18. 1074*s.* 12888*d.*
21. £967+£48 - 7 - 0
22. 9952*s.* 119432*d.*
23. 5764*s.*
24. 20000*s.*
25. 7434*s.* 89218*d.*
26. 852*s.* 10231*d.*
27. 661*s.* 6*d.*
28. 304*s.*
29. 260*s.*

AVOIRDUPOIS WEIGHT.

(Arithmetic, page 67.)

31. 191 qrs.
32. 543 qrs.
33. 142 qrs.
34. 1253 qrs.
35. 20 cwt. 80 qrs. 2240 lbs.
36. 859 qrs.
37. 377 qrs.
38. 1191 cwt. 4764 qrs.
39. 2049 qrs.
40. 14346 cwt.
41. 35714 qrs. 8928 cwt.
42. 62500 lbs. 2232 qrs.
43. 1459 qrs.
44. 3319 qrs. 829 cwt.

Work of some of these by RULE V., *page* 66.

31. 47 cwt. 3 qrs. 24 lbs.
564 84

32. 135 cwt. 3 qrs. 11 lbs.
1620 84

33. 35 cwt. 2 qrs. 19 lbs.
420 56

34. 313 cwt. 1 qr. 25 lbs.
3756 28

36. 214 cwt. 3 qrs. 0 lbs.
2568 84

37. 94 cwt. 1 qr. 11 lbs.
1128 28

TROY WEIGHT.

(Arithmetic, page 69.)

45. 232 dwts.
46. 43 oz.
47. 74 dwts.
48. 348 oz. 6960 dwts.

LONG MEASURE.

(Arithmetic, page 70.)

Exer.

50. 27 fur. 1080 perches
51. 753 fur.
53. 81987 per. 2049 fur.
54. 7818 yds. 1116 per. 27 fur.
55. 200 fur. 8034 per.

Exer.

44187 yards.

56. 83333 feet, 27777 yds. 5050 per. 126 fur.
57. 800 fur. 32000 per. 176000 yds. 528000 ft.

REDUCTION OF CLOTH MEASURE.

(Arithmetic, page 70.)

58. 115 qrs.
59. 1311 qrs.
60. 232 qrs.
61. 71 qrs.
62. 1872 qrs.; and 1872 ÷5=374 ells, 2 qrs.

REDUCTION OF SQUARE MEASURE.

(Arithmetic, page 70.)

63. $245 \times 30\frac{1}{4} = 7411\frac{1}{4}$ yds.; $7411\frac{1}{4} \times 9 = 66701\frac{1}{4}$ ft.
64. 1325419÷144=9204 ft. 43 in.

REDUCTION OF LAND MEASURE.

(Arithmetic, page 71.)

65. 937 roods.
66. 2345 roods.
67. 1024 roods, 40975 per.

REDUCTION OF TIME.

(Arithmetic, page 72.)

68. 408 hours
69. 28 hours, 1680 min.
70. 205761 min. 3429 hrs.
71. 183333 min. 3055 hrs.
72. 31−12=19 : then 19 + 30+31 + 30 + 31 + 31+28 + 31 + 24 =255
73. 31−8=23; and 23+ 29 +31 + 30 +31+ 30+31 + 31 +30 + 31+30+12=339
74. 31−17=14 : then 14 +30 + 31 + 30+31 +31 + 30 +31 +30 +25=283

MISCELLANEOUS EXERCISES.

(Arithmetic, page 72.)

Exer. 75. This will be solved most easily by dividing the following numbers successively by 5280, the number

of feet in a mile: viz. 41847426, 41707620, 139806 (their difference), and 41777523 (half their sum).

76. $700 \times 12 = 8400$ cuts; $8400 \times 120 = 1008000$ threads; $1008000 \times 2 = 2016000$ yards; also 1008000 half yards $= 504000$ yards: then $2016000 + 504000 = 2520000$ yards, the whole length; and this is to be reduced to miles in the common way.

77. $9 - 6\frac{1}{2} = 2\frac{1}{2}$ hours gained each day. Then, $365 \times 40 = 14600$, the days in 40 years. Also, $14600 \times 2 = 29200$ hours, and 14600 half hours $= 7300$ hours. Again, $29200 + 7300 = 36500$, the whole number of hours gained. Then, $36500 \div 12 = 3041$ days, 8 hours, and $3041 \div 365 = 8$ years, 121 days.

78. 655 hours, 39343 minutes; 42 hours, 2548 minutes, and 1358 minutes. Then, having obtained the first three answers of the question, divide the first of them successively by the second and third.

79. $237628 \div 31 = 7665\frac{13}{31}$ hours, or 319 days, $9\frac{13}{31}$ hours.

80. 91600000 British miles $= 483648000000$ feet. Divide this number by 1130, the number of feet travelled by sound in a second, and the quotient is $428007079\frac{73}{113}$, the number of seconds. Reducing this time to hours, &c., we have 118890 hours, 51 minutes, $19\frac{73}{113}$ seconds. Dividing 118890 by 6 we get 19815 quarter days, 0 hours. Dividing 19815 by 1461, the number of quarter days in a year, we get 13 years and a remainder 822 quarter days. Reducing 822 quarter days to days and hours, we get 205 days, 12 hours.

81. Add the two given numbers together, and multiply the sum by 640.

82. $576000000 \div 36 = 16000000$, the number of hours. Dividing this by 6 we get 2666666 quarter days and 4 hours. Dividing 2666666 by 1461, the number of quarter days in a year, we get 1825 years, 85 days, and 6 hours, which, with the former remainder of 4 hours, make up 10 hours in the answer.

83. 7532 minutes.

84. $328 \div 90$ gives 3 right angles, 58 degrees.

85. One radian $= 206265''$, which reduce to degrees and minutes.

86. $167° 43' = 10063'$, which divide by 3438, the number of minutes in a radian, approximately.

COMPOUND MULTIPLICATION.

(Arithmetic, page 81.)

Exer.		£	*s.*	*d.*
13.	Prodt. by	2= 3	15	7
	.....	7=13	4	6½
14.	.....	3= 2	0	1½
	.....	5= 3	6	10½
15.	.....	2= 0	5	5
	.....	4= 0	10	10
	.....	8= 1	1	8
16.	.....	2= 7	8	0½
	.....	3=11	2	0¾
	.....	6=22	4	1½
	.....	9=33	6	2¼
17.	.....	3= 3	15	9
	.....	7= 8	16	9
18.	.....	2= 1	8	2
	.....	11= 7	14	11
19.	.....	12= 7	18	0
	.....	8= 5	5	4
	.....	6= 3	19	0
	.....	4= 2	12	8
	.....	3= 1	19	6
	.....	2= 1	6	4
20.	.....	3= 1	17	9¾
	.....	9= 5	13	5¼
21.	.....	4=11	1	8
	.....	8=22	3	4
22.	.....	3= 5	9	6¾
	.....	11=20	1	8¼
23.	.....	12=23	12	0
	.....	6=11	16	0
	.....	9=17	14	0
	.....	3= 5	18	0
	.....	4= 7	17	4
24.	.....	6= 9	7	6
	.....	7=10	18	9
25.	.....	5= 2	1	10½
	.....	3= 3	15	4½
26.	.....	12=31	14	6
	.....	8=21	3	0

Exer.		£	*s.*	*d.*
	Prodt. by	6=15	17	3
	.....	4=10	11	6
27.	.....	7= 5	14	9¼
28.	.....	8= 1	10	10
	.....	7= 1	6	11¾
29.	.....	12= 4	3	3
	.....	10= 3	9	4½
	.....	6= 2	1	7½
	.....	5= 1	14	8¼
30.	.....	9=22	13	9
	.....	7=17	12	11
31.	.....	6=10	6	0
	.....	11=18	17	8
32.	.....	12= 2	13	3
	.....	8= 1	15	6
	.....	6= 1	6	7½
	.....	9= 1	19	11¼
33.	.....	8=10	14	6
	.....	10=13	8	1½
34.	.....	12=15	19	0
	.....	7= 9	6	1
35.	.....	9=13	16	0
	.....	10=15	6	8
36.	.....	12=21	11	9
	.....	8=14	7	10
37.	.....	10=18	11	8
38.	.....	12=10	3	6
	.....	9= 7	12	7½
39.	.....	10= 9	7	6
	.....	11=10	6	3
40.	.....	12= 2	16	3
	.....	10= 2	6	10½
41.	.....	11=32	14	6
42.	.....	12=21	19	6
	.....	11=20	2	10½
43.	.....	12=35	10	3
44.	.....	3= 4	17	3
	.....	5= 8	2	1

Exer.		£	s.	d.
	Prodt. by 15=	24	6	3
	..... 25=	40	10	5
45.	..... 4=	7	14	11
	..... 7=	13	11	1¼
	..... 16=	30	19	8
	..... 28=	54	4	5
46.	..... 4=	2	16	11
	..... 8=	5	13	10
	..... 16=	11	7	8
	..... 32=	22	15	4
	..... 64=	45	10	8
47.	..... 3=	3	7	9¾
	..... 7=	7	18	2¾
	..... 21=	23	14	8¼
	..... 49=	55	7	7¼
48.	..... 4=	2	5	6
	..... 6=	3	8	3
	..... 7=	3	19	7½
	..... 24=	13	13	0
	..... 28=	15	18	6
	..... 42=	23	17	9
49.	..... 5=	9	3	4
	..... 7=	12	16	8
	..... 25=	45	16	8
	..... 35=	64	3	4
50.	..... 4=	11	0	9
	..... 7=	19	6	3¾
	..... 28=	77	5	3
	..... 49=	135	4	2¼
51.	..... 12=	61	18	0
52.	..... 2=	4	17	3½
	..... 11=	26	15	1¼
53.	..... 4=	6	0	4
	..... 7=	10	10	7
54.	..... 10=	8	12	11
	..... 6=	5	3	9
	..... 3=	2	11	10½
	..... 5=	4	6	5½
55.	..... 12=	0	16	6
	..... 36=	2	9	6
	..... 2=	0	2	9
	..... 6=	0	8	3

Exer.		£	s.	d.
56.	Prodt. by 4=	2	5	3
	..... 10=	5	13	1½
57.	..... 9=	8	8	0
	..... 5=	4	13	4
58.	..... 12=	18	12	3
	..... 4=	6	4	1
	..... 8=	12	8	2
	..... 6=	9	6	1½
59.	..... 10=	23	4	2
	..... 5=	11	12	1
	..... 50=	116	0	10
	..... 2=	4	12	10
60.	..... 6=	3	18	1½
	..... 9=	5	17	2¼
	..... 54=	35	3	1½
61.	..... 8=	18	2	8
	..... 7=	15	17	4
	..... 56=	126	18	8
	..... 2=	4	10	8
62.	..... 8=	12	16	10
	..... 64=	102	14	8
63.	..... 6=	5	18	10½
	..... 11=	10	17	11¼
	..... 66=	65	7	7½
	..... 2=	1	19	7½
64.	..... 10=	6	7	1
	..... 7=	4	8	11½
	..... 70=	44	9	7
65.	..... 12=	1	11	0
	..... 72=	9	6	0
	..... 3=	0	7	9
	..... 9=	1	3	3
	..... 8=	1	0	8
66.	..... 11=	37	7	1
	..... 7=	23	15	5
	..... 77=	261	9	7
67.	..... 11=	11	8	8½
	..... 7=	7	5	6½
	..... 77=	80	0	11½
68.	..... 9=	6	9	9
	..... 81=	58	7	9
69.	..... 12=	4	0	0

Exer.		£	s.	d.
	Prodt. by 8¼=	28	0	0
	..... 3=	1	0	0
	..... 11=	3	13	4
	..... 88=	29	6	8
70.	..... 8=	13	8	0
	..... 11=	18	8	6
	..... 88=147		8	0
71.	..... 10=	11	12	3⅓
	..... 9=	10	9	0¾
	..... 90=104		10	7½
72.	..... 12=	21	16	0
	..... 8=	14	10	8
	..... 96=174		8	0
	..... 2=	3	12	8
73.	..... 12=	25	15	3
	..... 8=	17	3	6
	..... 96=206		2	0
74.	..... 12=	41	15	0
	..... 8=	27	16	8
	..... 96=334		0	0
	..... 2=	6	19	2
75.	..... 10=	7	15	7½
	..... 100=	77	16	3
	..... 3=	2	6	8¼
76.	..... 10=	5	13	9
	..... 100=	56	17	6
	..... 4=	2	5	6
77.	..... 12=	18	6	3
	..... 9=	13	14	8¼
	..... 108=164		16	3
	..... 3=	4	11	6¾
78.	..... 10=	22	18	4
	..... 11=	25	4	2
	..... 110=252		1	8
79.	..... 12=	0	9	3
	..... 108=	4	3	3
	..... 5=	0	3	10¼
	..... 10=	0	7	8½
	..... 110=	4	4	9½

Exer.		£	s.	d.
	Prodt. by 3=	0	2	3¾
80.	..... 12=	45	3	9
	..... 10=	37	13	1½
	..... 120=451		17	6
	..... 3=	11	5	11¼
81.	..... 11=	42	15	0¼
	..... 121=470		5	2¾
82.	..... 12=	31	17	3
	..... 10=	26	11	0½
	..... 120=318		12	6
	..... 5=	13	5	6¼
83.	..... 12=	18	8	0
	..... 11=	16	17	4
	..... 132=202		8	0
	..... 2=	3	1	4
84.	..... 12=	4	1	6
	..... 144=	48	18	0
	..... 5=	1	13	11½
85.	..... 12=	10	8	9
	..... 144=125		5	0
86.	..... 12=	9	7	0
	..... 144=112		4	0
	..... 6=	4	13	6
87.	..... 12=	26	16	6
	..... 144=321		18	0
	..... 11=	24	11	9½
88.	..... 12=	30	17	3
	..... 13=	33	8	8¼

		c.	q.	lbs.
89.	..... 3=	4	3	23
	..... 9=	14	3	13
90.	..... 7=	6	2	14
	..... 12=	11	1	12
	..... 84=	79	2	0
	..... 2=	1	3	16
91.	..... 12=	2	0	6
	..... 8=	1	7	0
	..... 96=	16	4	0

Exer.		£	*s.*	*d.*
92.	Price of first quantity ...	5	2	1
	..... second	5	10	3
	..... third	5	16	0
	..... fourth.....	9	8	7
	..... fifth	10	6	3
	..... sixth	10	18	9
	..... seventh	11	11	3
	..... eighth	12	3	9
	..... ninth	12	16	3
	..... tenth	13	8	9
	..... eleventh.....	14	1	3
	..... twelfth	13	6	4
	..... thirteenth	15	12	6

Ex.	*s.*	*d.*		£	*s.*	*d.*
93.	5	4×	10=	2	13	4
	5	4×	6=	1	12	0
	5	1×	12=	3	1	0
	5	1×	9=	2	5	9
	5	1×	108=27		9	0
	5	1×	4=	1	0	4
94.	Prodt. by		10=	3	3	9
	.....		11=	3	10	$1\frac{1}{2}$
	.....		110=35		1	3

Exer.			£	*s.*	*d.*
	Prodt. by	2=	0	12	9
95.	.....	10=	9	3	4
	.....	5=	4	11	8
	.....	50=45		16	8
	.....	2=	1	16	8
96.	.....	10=	5	4	2
97.	.....	9=	4	11	6
	.....	7=	3	11	2

COMPOUND DIVISION.

(Arithmetic, page 88.)

Exer.			£	*s.*	*d.*	
13.	Quotient by	2 =	36	19	1	
	.....	7 =	10	11	2	
14.	.....	3 =	2	14	6	
	.....	6 =	1	7	3	
15.	.....	7 =	5	15	$8\frac{1}{2}$	remainder $\frac{1}{2}d.$
16.	.....	4 =	9	2	11	
17.	.....	3 =	19	9	0	
	.....	4 =	14	6	9	
	.....	6 =	9	14	6	
	.....	9 =	6	9	8	
	.....	12 =	4	17	3	
18.	.....	3 =	19	17	$3\frac{3}{4}$	
	.....	9 =	6	12	$5\frac{1}{4}$	

Exer.			£	s.	d.		
19.	Quotient by	5 =	9	13	$7\frac{1}{5}$,	remainder	1*d.*
	.....	10 =	4	16	$9\frac{3}{4}$,	.....	1*d.*
20.	.....	4 =	2	8	8		
	.....	8 =	1	4	4		
21.	.....	12 =	2	11	$10\frac{1}{2}$,	.....	$2\frac{1}{2}$*d.*
	.....	8 =	3	17	10,	.....	$\frac{1}{2}$*d.*
	.....	6 =	5	3	$9\frac{1}{4}$,	.....	1*d.*
	.....	4 =	7	15	8,	.....	$\frac{1}{2}$*d.*
22.	.....	9 =	5	11	$1\frac{1}{4}$,	.....	$\frac{3}{4}$*d.*
	.....	7 =	7	2	$10\frac{1}{4}$,	.....	$\frac{1}{4}$*d.*
23.	.....	7 =	7	18	$3\frac{1}{2}$,	.....	$\frac{1}{4}$*d.*
	.....	11 =	5	0	$8\frac{3}{4}$,	.....	$\frac{1}{2}$*d.*
24.	.....	9 =	5	19	2,	.....	$1\frac{1}{2}$*d.*
25.	.....	12 =	9	9	$6\frac{3}{4}$		
	.....	8 =	14	4	4,	.....	1*d.*
26.	.....	12 =	4	9	$2\frac{1}{4}$		
	.....	10 =	5	7	$0\frac{1}{2}$,	.....	1*d.*
27.	.....	10 =	3	3	$0\frac{1}{2}$,	.....	$1\frac{1}{2}$*d.*
	.....	7 =	4	10	$0\frac{1}{2}$,	.....	$\frac{1}{2}$*d.*
28.	.....	8 =	4	16	3		
	.....	11 =	3	10	0		
29.	.....	12 =	10	8	$0\frac{1}{3}$		
	.....	7 =	17	16	$7\frac{1}{3}$,	.....	$1\frac{1}{2}$*d.*
30.	.....	11 =	9	1	$9\frac{3}{4}$,	.....	$\frac{3}{4}$*d.*
31.	.....	12 =	31	5	$10\frac{1}{2}$		
32.	.....	3 =	46	5	0		
	.....	5 =	27	15	0		
	.....	10 =	13	17	6		
	.....	15 =	9	5	0		
	.....	30 =	4	12	6		
	.....	50 =	2	15	6		
33.	.....	3 =	37	0	10		
	.....	6 =	18	10	5		
	.....	9 =	12	6	$11\frac{1}{4}$,	.....	$\frac{3}{4}$*d.*
	.....	10 =	11	2	3		
	.....	18 =	6	3	$5\frac{1}{2}$,	.....	3*d.*
	.....	30 =	3	14	1		
	.....	60 =	1	17	$0\frac{1}{2}$		
	.....	90 =	1	4	$8\frac{1}{4}$,	.....	$7\frac{1}{2}$*d.*
34.	.....	5 =	51	3	9		
	.....	9 =	28	8	9		
	.....	45 =	5	13	9		

Exer.		£	s.	d.	
35. Quotient by	5 =	149	4	0	
.....	11 =	67	16	$4\frac{1}{4}$,	remainder $1\frac{1}{4}d.$
.....	55 =	13	11	$3\frac{1}{4}$,	..... $1\frac{1}{4}d.$

In most of the remaining solutions in this rule, the respective remainders, after the finding of the pounds, &c., of the answers, are given.

(Arithmetic, page 89.)

Exer.

36. £9, or 180*s.*; 11*s.*, or 132*d.*
37. £9 - 3, or 183*s.*; 13*s.* 7*d.*, or 163*d*; 10*d.*, or 40*f.*
38. £12 - 4, or 244*s.*; 16*s.* 2*d.*, or 194*d.*
39. £21, or 420*s.*; 4*s.*, or 48*d.*; 22*d.*, or 88*f.*
40. £4 - 12, or 92*s.*; 18*s.* 6*d.*, or 222*d.*
41. £7 - 9, or 149*s.*; 20*s.* 4*d.*, or 244*d.*; $29\frac{1}{2}d.$, or 118*f.*
42. £31 - 6, or 626*s.*; 14*s.* 11*d.*, or 179*d.*; 26*d.*, or 104*f.*
43. £54 - 13, or 1093*s.*; 53*s.* 2*d.*, or 638*d.*; 53*d.*, or 212*f.*
44. £37 - 18, or 758*s.*; 48*s.* 1*d.*, or 577*d.*; $9\frac{1}{2}d.$, or 38*f.*
45. £82 - 16, or 1656*s.*; 41*s.*, or 492*d.*; 67*d.*, or 268*f.*
46. £53 - 12, or 1072*s.*; 71*s.*, or 852*d.*; 33*d.*, or 132*f.*
47. £35 - 14, or 714*s.*; 96*s.*, or 1152*d.*; 19*d.*, or 76*f.*
48. £46 - 6, or 926*s.*; 128*s.*, or 1536*d.*; 73*d.*, or 292*f.*
49. £197 - 2, or 3942*s.*; 123*s.* 4*d.*, or 1480*d.*; 73*d.*, or 292*f.*
50. £29 - 11, or 591*s.*; 278*s.* 4*d.*, or 3340*d.*; $210\frac{1}{2}d.$, or 842*f.*
51. £35, or 700*s.*; 335*s.*, or 4020*d.*; 5*d.*, or 20*f.*
52. £4571 - 13, or 91433*s.*; 1758*s.* 4*d.*, or 21100*d.*
53. £2045 - 16, or 40916*s.*; 86*s.* 5*d.*, or 1037*d.*; $1037\frac{1}{4}d.$, or 4149*f.*
54. £1288 - 2, or 25762*s.*; 169*s.* 8*d.*, or 2036*d.*; $689\frac{1}{2}d.$, or 2758*f.*
55. £1501 - 16, or 30036*s.*; 932*s.* 8*d.*, or 11192*d.*; 278*d.*, or 1112*f.*
56. Quotient by 12=£1 - 6 - 8; by 8=£2.
57. Divide by 8 to find the price of a stone; and divide the quotient by 7 and the result by 2 to find the price of a pound.
58. £16 - 11, or 331*s.*; 19*s.* 6*d.*, or 234*d.*
59. £45 - 5, or 905*s.*; 5*s.*, or 60*d.*; 240*f.*
60. In dividing by 365, the remainders are £200, or 4000*s.*; 350*s.*, or 4200*d.*; 185*d.*, or 740*f.*

Exer.

53. As 365 days, 6 hours, 9 min. 10 sec. : 1 hour :: 574000000 miles ; or, as 31558150 seconds : 3600 seconds :: 574000000 miles : $65479\frac{3896}{3155815}$ miles.
54. As 1 oz. : 9 oz. 13 dwt. 8 grs. :: 7*s.* 6*d.* ; or, as 480 grs. : 4640 grs. :: 7*s.* 6*d.* : £3 - 12 - 6.
55. From May 1 till December 18, there are 231 days. (*See Arithmetic, page* 72). Then, as 365 days : 231 days :: £89 - 12 - 6 ; or, as 365 days : 231 days :: 21510*d.* : $13613\frac{65}{365}d.$, or £56 - 14 - $5\frac{13}{73}$. After the reduction, the first and third terms might have been divided by 5.
56. As 59 ac. 3 ro. 20 po. : 12 ac. 2 ro. 30 po. :: £134 - 4 - 0 ; or, as 9580 po. : 2030 po. :: £134 - 4 - 0 : £28 - 8 - $8\frac{118}{479}$.
57. As 12 : 365 :: £2 - 13 - 6 : £81 - 7 - $3\frac{1}{2}$.
58. As 20 : 311 :: £37 - 12 - 6 : £585 - 1 - $4\frac{1}{4}$.

MEASURES AND MULTIPLES.

(Arithmetic, page 137.)

17. Reject 6, as it is contained in 18.
19. Reject 8 and 12. Then divide by 4 or 5, and reject one of the quotients.
20. Reject 7 and 12, and divide the rest by 21.
21. Divide by 9 ; reject one of the quotients, &c.
22. Reject 2, 3, 4 ; divide by 2 or 3, and reject one of the quotients.
23. Reject 8 ; divide by 2, and reject 7.
24. Divide by 11, 8, and 17.
25. Divide by 23 and 37.

REDUCTION OF FRACTIONS.

(Arithmetic, page 141.)

19. In finding the common denominator, reject 12 and 8, divide by 3 or 8, and reject one of the quotients.
20. Reject 6, 5, 9 ; and divide by 3.
21. Reject 9 and one 60.
24. Reject 12 and 16, and divide by 6.
25. Reject 100 and 1000.
28. Reject all the denominators, except the last.

ADDITION OF FRACTIONS.

(Arithmetic, page 144.)

Exer.

1. Com. denom. 60; num. 32 and 27
2. .... 216; ..... 189, 18, 204, 207, and 160
3. ..... 144; ..... 126, 84, 117, 88, and 114
4. ..... 128; 64, 32, 16, 8, 4, 2, 1
5. ..... 128; 64, 96, 112, 120, 124, 126, 127
6. ..... 420; 280, 315, 336, 350, 360
7. ..... 1320; 495, 1232, 528, 540
8. ..... 168; 105, 28, 166
9. ..... 40; 24, 35, 12
10. .... 672; 224, 384, 441
11. ..... 168; 105, 60, 28
12. ..... 180; 72, 115, 108
13. ..... 756; 189, 260
14. ..... 264; 99, 12, 77, 87
15. ..... 180; 24, 63, 15, 50
16. ..... 84; 63, 28, 48, 63
17. ..... 600; 552, 240, 175, 440

SUBTRACTION OF FRACTIONS.

(Arithmetic, page 149.)

Exer.

5. Com. denom. 126; num. 111, 13
6. ..... 42; 15, 26
7. ..... 16; 12, 13
8. ..... 77; 11, 7
9. ..... 140; 115, 108
10. ..... 48; 2, 3
11. ..... 105; 80, 56
12. ..... 165; 27, 11
13. ..... 864; 171, 172
14. ..... 225; 33, 35
15. .... 224; 161, 64
16. ..... 198; 132, 154, 171

In this exercise add the first and second numbers together, and take the third from the sum.

17. Com. denom. 26; num. 95, 68
18. ..... 182; 1677, 728
19. ..... 26; 115, 104
20. ..... 68; 51, 204

Exer.

21. Com. denom. 58; num. 174, 29
22. ..... 48; 63, 52

FRACTIONAL MULTIPLICATION AND DIVISION.

Group I.

(Arithmetic, page 173.)

Exer.

8. The improper fractions are $\frac{15}{8}$ and $\frac{32}{17}$.
9. ..... $\frac{124}{36}$ and $\frac{232}{49}$.
10. ..... $\frac{131}{6}$, $\frac{152}{7}$, and $\frac{173}{8}$.
13. ..... $\frac{132}{10}$ and $\frac{25}{14}$.
18. ..... $\frac{65}{28}$ and $\frac{66}{65}$. Here, reject the numerator 65, and the same denominator, and the answer is $\frac{66}{28}$, or $2\frac{5}{14}$.
23. Multiply the first by the second, and the third by the fourth, and add the products, $\frac{7}{18}$ and $\frac{7}{30}$, together.

Group II.

(Arithmetic, page 175.)

32. The improper fractions are $\frac{19}{4}$ and $\frac{41}{8}$.
33. ..... $\frac{381}{20}$ and $\frac{31}{15}$.
35. ..... $\frac{42}{5}$ and $\frac{26}{3}$.
40. ..... $\frac{50}{17}$ and $\frac{53}{18}$.
41. ..... $\frac{27}{20}$ and $\frac{62}{55}$.
42. ..... $\frac{1}{1}$.
44. ..... $\frac{70}{9}$ and $\frac{8}{1}$.
45. ..... $\frac{9}{2}$ and $\frac{15}{1}$.
48. By reducing the compound fractions, this becomes $\frac{8}{15} \div \frac{9}{14}$.
51. The improper fractions are $\frac{32}{9}$ and $\frac{21}{8}$.
53. By reducing the compound fractions, this becomes $\frac{3}{4} \div \frac{32}{45}$.
54. Divide the first by the second, and the third by the fourth; and subtract the latter quotient, $\frac{11}{16}$, from the former, $\frac{14}{15}$.
55. Divide as in the last, and add the quotients, $\frac{7}{10}$ and $1\frac{19}{25}$, together.

Exer.

56. $365 = \frac{1460}{4}$, and $365\frac{1}{4} = \frac{1461}{4}$. Divide the first numerator by the second.

57. The improper fractions are $\frac{360}{49}$ and $\frac{120}{7}$. Divide the numerators by 120, and the denominators by 7, &c.

Group III.

(Arithmetic, page 178.)

62. Multiply £89 - 14 - 6 by 17; divide by 4. *Ans.* £381 - 6 - $7\frac{1}{2}$.

63. £56 - 10 - 4 ÷ $\frac{17}{3}$ = £56 - 10 - 4 × $\frac{3}{17}$ = £9 - 19 - $5\frac{11}{17}$ *Answ.* This may now be proved by adding together 5 of this, that is, £49 - 17 - $4\frac{4}{17}$ and $\frac{2}{3}$ of it, £6 - 12 - $11\frac{13}{17}$.

64. £77 - 3 - 5 ÷ 5 = £15 - 8 - $8\frac{1}{5}$ *Answ.*

65. Multiply £98 - 14 - 6 by 17 and divide by 3.

66. Reduce $\frac{1}{4}$ of $3\frac{1}{2}$ to the fraction $\frac{7}{8}$. £3 10 - 6 × $\frac{7}{8}$ ÷ $\frac{7}{8}$ = £3 - 10 - 6.

67. $\frac{5}{7} \div 1\frac{3}{4} = \frac{5}{7} \div \frac{7}{4} = \frac{5}{7} \times \frac{4}{7} = \frac{20}{49}$ *Answ.*

68. $\frac{7}{9} \div 1\frac{3}{4} = \frac{4}{9}$ *Answ.*

69. $\frac{3}{10} \div 4\frac{1}{16} = \frac{24}{325}$ *Answ.*

70. $\frac{86}{27} \div \frac{20}{63} = \frac{301}{30} = 10\frac{1}{30}$ *Answ.*

71. $62\frac{3}{8} \div 2\frac{5}{6} = 22\frac{1}{68}$ minutes *Answ.* The improper fractions are $\frac{499}{8} \div \frac{17}{6} = \frac{1497}{68}$ minutes.

72. $26\frac{5}{8} \div 15\frac{43}{60} = 1\frac{1309}{1886}$ *Answ.* The improper fractions are $\frac{213}{8} \div \frac{943}{60} = \frac{31905}{1886}$.

73. Dividing by 25 we get $\frac{2}{4}$, the inverse of which is $\frac{4}{2}$, or $1\frac{1}{2}$.

74. The improper fraction is $\frac{13}{5}$, the reciprocal of which is $\frac{5}{13}$.

75. As $2\frac{3}{5}$ shares : 1 share :: £585 - 8 - 6 : £225 - 3 - $3\frac{3}{13}$.

76. $25 \times \frac{3}{5} = 15$. $15 \times \frac{10}{21} = 7\frac{1}{7}$. $7\frac{1}{7} \div \frac{3}{100} = 238\frac{2}{21}$ *Answ.*

77. $34 \times \frac{5}{8} = 21\frac{1}{4}$. $21\frac{1}{4} \times \frac{10}{17} = 12\frac{1}{2}$. $12\frac{1}{2} \div \frac{7}{1000} = 1785\frac{5}{7}$.

78. $35 \div 84 = \frac{5}{12}$ *Answ.*

79. $84\frac{3}{4} \div 35\frac{1}{2} = 2\frac{55}{142}$ *Answ.*

80. $2\frac{1}{5} \div 11 \times 20 = 4$ sheep *Answ.*

81. $\frac{1}{4}$ of a score = 5 *Answ.*

82. $\frac{1}{4}$ of a score = 5 *Answ.*

83. $\frac{1}{6}$ dozen = 2 *Answ.*

84. 5 doz. ÷ 3 = $1\frac{2}{3}$ doz. $1\frac{2}{3}$ doz. ÷ 5 = 4 eggs.

85. Reducing to improper fractions $\frac{11}{4} \div \frac{33}{8} = \frac{2}{3}$ *Answ.*

86. Reducing to improper fractions $\frac{11}{4} \times \frac{33}{8} = \frac{363}{32}$ or $11\frac{11}{32}$.

Exer.

87. *Answ.* $\frac{9}{20}$.
88. *Answ.* $\frac{15}{48}$.
89. *Answ.* $\frac{8}{9}$.
90. $9\frac{1}{2} \div 90\frac{1}{2}$ gives $\frac{19}{181}$ lbs. *Answ.*
91. $9\frac{1}{2}$ lbs. $\div 100 = \frac{19}{200}$ lbs. tin ; $90\frac{1}{2}$ lbs. $\div 100 = \frac{181}{200}$ lbs. copper.
92. $\frac{100}{328}$ Metre.

NOTATION AND NUMERATION OF DECIMAL FRACTIONS.

(Arithmetic, page 183.)

Notation.

Exer.	*Exer.*
1. ·0306	6. ·00057
2. ·00003	7. ·0000200, or ·00002
3. ·0000001	8. 5·09
4. ·0001	
5. ·1	

Numeration.

Exer.

1. Fifty-eight hundredths.
2. One hundred and six thousandths.
3. Seven thousandths.
4. Seven ten-thousandths.
5. Thirty-one, and seventy-three hundredths.
6. Three, and one hundred and seventy-three thousandths.
7. Four hundred-thousandths.
8. Forty-one hundred-thousandths.
9. Thirty-four thousand, five hundred, and sixty-seven millionths.
10. One ten-millionth.

REDUCTION OF DECIMAL FRACTIONS.

(Arithmetic, page 189.)

Exer.

17. 10*s.* 9*d.* = 129*d.* = $£\frac{129}{240} = £\frac{43}{80} = £·5375$
18. $10\frac{1}{2}$ = 21 halfpence = $£\frac{21}{480} = £\frac{7}{160} = £·04375$
19. 17*s.* 7*d.* = 211*d.* = $£\frac{211}{240} = £·87916$

Exer.

20. 3 roods, 11 per.=131 per.=$\frac{131}{160}$ acre=·81875 acre.
21. 2 qrs. 8 lbs.=64 lbs.=$\frac{64}{112}$ cwt.=$\frac{4}{7}$ cwt.=·571428 cwt.
22. 37 per.=$\frac{37}{320}$ mile=·115625 mile.
23. $3\frac{1}{2}$ hours=7 half-hours=$\frac{7}{48}$ day=·14583 day.
24. $15\frac{1}{2}$ min.=31 half-minutes=$\frac{31}{1920}$ hour=·2583 hr.
25. 3 cwt. 1 qr. 7 lbs.=371 lbs.=$\frac{371}{2240}$ ton=$\frac{53}{320}$ ton =·165625 ton.
26. $6\frac{1}{2}d.$=13 halfpence=$\frac{13}{24}s.$=·5416$s.$
29. The quarter of wheat is 4 cwt. 8 lbs., or $4\frac{1}{14}$ cwt., or 4·0714285 cwt.
30. 5 hours, 48 minutes, 49·7 seconds=20929·7 seconds, and 24 hours=86400 seconds. Then, dividing the former by the latter, we get ·2422419, nearly.
31. ·0675 cwt.=·27 qr.=7·56 lbs.=$7\frac{56}{100}$ lbs.=$7\frac{14}{25}$ lbs.
32. ·4625 ton=9·25 cwt.=9 cwt. 1 qr.
33. £·0484=·968$s.$=11·616$d.$=$11\frac{616}{1000}d.$=$11\frac{77}{125}d.$
34. ·8845 acre=3·538 roods=3 roods, 21·52 perches =3 roods, $21\frac{13}{25}$ perches.
35. ·00213 day=·05112 hour=3·0672 minutes=3 minutes, 4·032 seconds=3 min. $4\frac{4}{125}$ seconds.
36. £·7777=15·5555$s.$=15$s.$ 6·6666$d.$=15$s.$ $6\frac{2}{3}d.$ Otherwise:—£·7=£$\frac{7}{9}$=15$s.$ $6\frac{2}{3}d.$
37. ·285714 cwt.=1·142857 qr.=1 qr. 3·9 lbs.=1 qr. 4 lbs.
38. ·1136 mile=·9090 furlong=36·3636 perches=36 perches, 1·9 yard=36 perches, 2 yards
39. ·615$s.$=7·38$d.$=$7\frac{19}{50}d.$
40. £·4835=9·670$s.$=9$s.$ 8·048$d.$
41. ·2383 degree=14·29 minutes=14·3 minutes=14 minutes, 18 seconds.
42. £·06=1·3$s.$=1$s.$ 3·9$d.$=1$s.$ 4$d.$
43. ·47916 lbs.=5·749 oz.=5·75 oz.=5 oz. 15 dwt.
44. £·428=8·56$s.$=8$s.$ 6·78$d.$=8$s.$ $6\frac{3}{4}d.$, nearly.
45. ·4375$s.$=5·25$d.$=$5\frac{1}{4}d.$
46. ·09375 acre=0·375 rood=15 perches.
47. ·4 foot=5·3 inches=$5\frac{1}{3}$ inches.
48. ·530588715 day × 24=12·73412916 hours; ·73412916 hour × 60=44·0477496 minutes; and ·0477496 minute × 60=2·864976 seconds.

ADDITION OF DECIMAL FRACTIONS.

(Arithmetic, page 192.)

Exer. 1.

1·83
5·674
·3125
18·3
100
38·62
4·3957
·5

169·6322

Exer. 2.

93·617843
7·836
12·25
·71375
4·391
7·839
3·7674285
·8693

131·2843215

Exer. 3.

51·25
3·4444444444444
·6373737373737
7·8855555555556*
7·875
7·8758758758759
11·1111111111111

90·0793607243607

Exer. 4.

·7354
·7354444444444
·7354545455
·7354735473 55
·07354
·073545454545

3·088857991799

Exer. 5.

·3
·333333333333
·45
·4545454545 45
·351351351351
·6468
·646888888889
·646868686869
·646846846847

4·476634567834

Exer. 6.

·888888889
·878787879
·876876876

2·644553644

Exer. 7.

·5
·75
·875
·9375
·96875
·984375
·9921875

6·0078125

Exer. 8.

·6666666667
·75
·8
·8333333333
·8571428571

3·9071428571

Exer. 9.

·33333333333333
·57142857142857
·65625

1·56101190476190

* This figure is made 6 instead of 5, as being nearer the truth. The like is done in many other cases.

SUBTRACTION OF DECIMAL FRACTIONS.

(Arithmetic, page 193.)

Exer. 1.	*Exer.* 2.	*Exer.* 3.	*Exer.* 4.	*Exer.* 5.	*Exer.* 6.
3·468	6·45	56·429	34·528	·682	13·6666
1·2591	1·345455	29·6853	10·6347	·09647	4·345
2·2089	5·104$\dot{5}$45	26·7437	23·8933	·58553	9·321$\dot{6}$

Exer. 7.	*Exer.* 8.	*Exer.* 9.	*Exer.* 10.
5·83	17·4444	3·34234234	8·375
4·17172	·4848	1·75757576	7·42857142857
1·65$\dot{8}$28	16·95$\dot{9}$6	1·5$\dot{8}$476$\dot{6}$58	0·946$\dot{4}$2857$\dot{1}$43

Exer. 11.	*Exer.* 12.
7·06944444444	15·0588235294117647058$\dot{8}$
4·10769230769	13·24
2·961$\dot{7}$5213$\dot{6}$75	1·81$\dot{8}$8235294117647058$\dot{8}$

MULTIPLICATION OF DECIMAL FRACTIONS.

(Arithmetic, page 194.)

Exer. 5. 1·05 × 1·05 = 1·1025, and 1·1025 × 1·05 = 1·157625.

Exer. 7. ·1 × ·1 = ·01 ; ·01 × ·1 = ·001 ; and ·001 × ·1 = ·0001.

Exer. 13.	*Exer.* 14.	*Exer.* 15.	*Exer.* 16.	*Exer.* 17.
1·1236740*	7·28571429	24·636364	·863541	·1347866
476321·1	50447·63	7432·	38901·	397882·
11236740	2185714287	49272728	86354	2695732
1123674	437142857	7390909	7772	1078293
224735	51000000	985454	691	107829
33710	2914286	172454	26	9435
6742	291428	5·7821$\dot{5}$45	·094843	1213
786	3643			40
45	267·7066501			·03892542
1·2626432				

* In this exercise, one figure more than the required number is wrought for, to attain greater accuracy; and the same is done in the rest. In such cases, the last figure thus found may be then rejected; and if it be above 5, the figure before it should be increased by unity.

Exer 18.

·26736
85782·

5347
2139
187
13
2

·07688, or
·0769

Exer. 19.

2·6564190
7327327·1

26564190
18594933
531284
79692
18595
531
80
19

4·5789324

Exer. 20.

1·656566
4848484·1

1656566
662626
132525
6626
1325
66
13

2·459747, or
2·45975

Exer. 21.

·053497
621740·

21399
3745
53
11

·0025208, or
·002521

Exer. 22.

·6428571
6074671·

642857
450000
38571
2571
450

·1134449, or
·113445

Exer. 23.

·851852
5087845·

4259260
340741
68148
5963
684

·4674797, or
·46748

Exer. 24.

2·6923077
9888888·

21538462
2153846
215385
21538
2154
215
23

2·3931623

Exer. 25.

7925·648
454·111

7925648
792565
79256
31702
3963
317

883345·1

Exer. 26.

7925·64800
571289·92

1585129600
713308320
71330832
6340518
158513
7926
5548
396

237628·1653

Exer. 27. 1·4375 cwt. × 27·9 = 40·10625 cwt., or 2 tons, 0 cwt. 0 qrs. 11·9 lbs.

DIVISION IN DECIMAL FRACTIONS.

(Arithmetic, page 199.)

In the following solutions, the numbers are prepared for division; and, in a few of the more difficult, the whole operation is performed by the contracted method. In those in which a cipher is placed in the quotient, a cipher or a periodical figure is to be immediately annexed to the dividend, except in exercise 12, where the figure 1 is annexed to complete the period.

Exer.

1. 26175) 47580(
2. 84736) 3412 (0·
3. 3365) 468777 (
4. 77482) 585 (0·
5. 3829) 753470 (
6. 706249) 655555 (0·
7. 10473654) 1000000 (0·
8. 3738) 750 (0·
9. 62222) 50909 (0·
10. 428571) 625000 (
11. 230769) 90909 (0·

Exer.

12. 3125) 21621 (0·
13. 834970) 79085 (0·
14. 13543516) 616161 (0·
15. 37538) 23600000 (
16. 531000) 7126491 (
17. 897000) 879454 (0·
18. 52734567) 52737373 (
19. 4923076) 2370370 (0·
20. 104869) 35493600 (
21. 125172) 10000000 (

Exer. 6.—706249) 6555555 (·928222, nearly.

6356241

199314

141250

58064

56500

1564

1412

152

141

11

Exer. 7. 10473654) 100000000 (·09547766
94262886
5737114
5236827
500287
418946
81341
73315
8026
7331
695
628
67

Exer. 14. 13543516) 61616162 (·04549495
54174064
7442098
6771758
670340
541741
128599
121892
6707
5417
1290
1219
71

Exer. 18. 52734567) 52737373 (1·000053224
52734567
2806737
2636728
170009
158204
11805
10547
1258
1055
203

PRACTICE.

(Arithmetic, page 205.)

Exer.		£	s.	d.
9.	Parts, 2*s.* 6*d.* and $2\frac{1}{2}d.$			
	Price at 2*s.* 6*d.*	= 203	2	6
	..... $2\frac{1}{2}d.=\frac{1}{12}$ of 2*s.* 6*d.* ...	= 16	18	$6\frac{1}{2}$
10.	..... 2*s.* 6*d.*	= 178	12	6
	..... 3*d.*	= 17	17	3
11.	Parts, 5*s.*, 1*s.*, 10*d.* ($=\frac{1}{6}$ of 5*s.*); or 5*s.*, 1*s.* 8*d.* ($=\frac{1}{3}$ of 5*s.*), 2*d.*			
	Price at 5*s.*	= 493	5	0
	..... 1*s.*	= 98	13	0
	..... 10*d.*	= 82	4	2
	..... 1*s.* 8*d.*	= 164	8	4
	..... 2*d.*	= 16	8	10
12.	Parts, 4*s.*, 1*s.* 8*d.* ($=£\frac{1}{12}$).			
	Price at 4*s.*	= 149	16	0
	..... 1*s.* 8*d.*	= 62	8	4
13.	Parts, 4*s.*, 6*d*, 3*d.*, $1\frac{1}{2}d.$; or 4*s.*, 8*d.*, 2*d.*, $\frac{1}{2}d.$			
	Price at 4*s.*	= 337	16	0
	..... 6*d.*	= 42	4	6
	..... 3*d.*	= 21	2	3
	..... $1\frac{1}{2}d.$	= 10	11	$1\frac{1}{2}$
	..... 8*d.*	= 56	6	0
	..... 2*d.*	= 14	1	6
	..... $\frac{1}{2}d.$	= 3	10	$4\frac{1}{2}$
14.	Parts, 2*s.*, 16*s.* ($=2s.\times 8$), 6*d.*; or 10*s.*, 4*s.*, 4*s.*, 6*d.*			
	Price at 2*s.*	= 247	12	0
	..... 16*s.*	=1980	16	0
	..... 6*d.*	= 61	18	0
	..... 10*s.*	=1238	0	0
	..... 4*s.*	= 495	4	0
15.	Parts, 10*s.*, 1*s.* 8*d.* ($=\frac{1}{6}$ of 10*s.*)			
	Price at 10*s.*	=2963	0	0
	..... 1*s.* 8*d.*	= 493	16	8
16.	Parts, 6*s.* 8*d.*, 2*s.*; or 4*s.*, 4*s.*, 8*d.*			
	Price at 6*s.* 8*d.*	= 104	6	8
	..... 2*s.*	= 31	6	0
	..... 4*s.*	= 62	12	0
	..... 8*d.*	= 10	8	8

Exer.		£	*s.*	*d.*
17.	Price at 5*s.*	=1483	10	0
	..... 10*d.*	= 247	5	0
18.	Parts, 10*s.*, 1*s.* 3*d.* (=$\frac{1}{8}$ of 10*s.*), 1½*d.* (=$\frac{1}{10}$ of 1*s.* 3*d.*); or 10*s.*, 1*s.*, 3*d.*, 1½*d.*			
	Price at 10*s.*	=1788	0	0
	..... 1*s.* 3*d.*	= 223	10	0
	..... 1½*d.*	= 22	7	0
	..... 1*s.*	= 178	16	0
	..... 3*d.*	= 44	14	0
19.	Parts, 10*s.*, 6*s.* 8*d.*, 2*s.*; or 10*s.*, 4*s.*, 4*s.*, 8*d.*			
	Price at 10*s.*	= 479	0	0
	..... 6*s.* 8*d.*	= 319	6	8
	..... 2*s.*	= 95	16	0
	..... 4*s.*	= 191	12	0
	..... 8*d.*	= 31	18	8
20.	Parts, 5*s.*, 1*s.* 8*d.*, 2½*d.* (=$\frac{1}{8}$ of 1*s.* 8*d.*)			
	Price at 5*s.*	= 474	10	0
	..... 1*s.* 8*d.*	= 153	3	4
	..... 2½*d.*	= 19	15	5
21.	Parts, 10*s.*, 2*s.* 6*d.*, 1*s.*; or 10*s.*, 2*s.*, 1*s.*, 6*d.*			
	Price at 10*s.*	= 797	0	0
	..... 2*s.* 6*d.*	= 199	5	0
	..... 1*s.*	= 79	14	0
	..... 2*s.*	= 159	8	0
	..... 6*d.*	= 39	17	0
22.	Parts, 10*s.*, 4*s.*, 10*d.* (=$\frac{1}{12}$ of 10*s.*)			
	Price at 10*s.*	= 347	10	0
	..... 4*s.*	= 139	0	0
	..... 10*d.*	= 28	19	2
23.	..... 4*s.*	= 477	4	0
	..... 2*s.* 6*d.*	= 273	5	0
24.	Parts, 6*s.* 8*d.*, 1*s.*; or 5*s.*, 2*s.*, 8*d.*			
	Price at 6*s.* 8*d.*	= 241	13	4
	..... 1*s.*	= 36	5	0
	..... 5*s.*	= 181	5	0
	..... 2*s.*	= 72	10	0
	..... 8*d.*	= 24	3	4
25.	..... 10*s.*	= 294	10	0
	..... 1*s.*	= 29	9	0
	..... 6*d.*	= 14	14	6

Exer.			£	s.	d.
26.	Parts, 10*s.*, 1*s.* 8*d.* (=$\frac{1}{6}$ of 10*s.*), 5*d.*; or 6*s.* 8*d.*, 5*s.*, 5*d.* (=$\frac{1}{12}$ of 5*s.*)				
	Price at 10*s.*	=	143	0	0
	..... 1*s.* 8*d.*	=	23	16	8
	..... 5*d.*	=	5	19	2
	..... 6*s.* 8*d.*	=	95	6	8
	..... 5*s.*	=	71	10	0
27.	Parts, 3*d.*, 2*d.*, $\frac{3}{4}$*d.* (=$\frac{1}{4}$ of 3*d.*)				
	Price at 3*d.*	=	1912	3	
	..... 2*d.*	=	1274	10	
	..... $\frac{3}{4}$*d.*	=	478		$0\frac{3}{4}$
28.	..... 6*d.*	=	2864	0	
	..... 1*d.*	=	477	4	
	..... $\frac{1}{4}$*d.*	=	119		4
29.	Parts, 6*d.*, 4*d.*, $\frac{3}{4}$*d.* (=$\frac{1}{8}$ of 6*d.*)				
	Price at 6*d.*	=	3425	6	
	..... 4*d.*	=	2163	8	
	..... $\frac{3}{4}$*d.*	=	405		$8\frac{1}{4}$
30.	..... 3*d.*	=	247	9	
	..... $\frac{1}{2}$*d.*	=	41		$3\frac{1}{2}$
31.	Parts, 10*s.*, 6*s.* 8*d.*, 1*s.* (=$\frac{1}{10}$ of 10*s.*)				
	Price at £4	=	1744	0	0
	..... 10*s.*	=	218	0	0
	..... 6*s.* 8*d.*	=	145	6	8
	..... 1*s.*	=	21	16	0
32.	Parts, 5*s.*, $7\frac{1}{2}$*d.* (=$\frac{1}{8}$ of 5*s.*), $\frac{3}{4}$*d.* (=$\frac{1}{10}$ of $7\frac{1}{2}$*d.*); or 5*s.*, 6*d.*, 2*d.*, $\frac{1}{4}$*d.*				
	Price at 5*s.*	=	931	5	0
	..... $7\frac{1}{2}$*d.*	=	116	8	$1\frac{1}{2}$
	..... $\frac{3}{4}$*d.*	=	11	12	$9\frac{3}{4}$
	..... 6*d.*	=	93	2	6
	..... 2*d.*	=	31	0	10
	..... $\frac{1}{4}$*d.*	=	3	17	$7\frac{1}{4}$
33.	Multiply by 5, and take $\frac{1}{12}$ for 1*d.*				
34.	Parts, 10*s.*, 5*s.*, 1*s.* 8*d.* (=$\frac{1}{8}$ of 5*s.*), $2\frac{1}{2}$*d.* (=$\frac{1}{8}$ of 1*s.* 8*d.*)				
	Price at £2	=	15826	0	0
	..... 10*s.*	=	3956	10	0
	..... 5*s.*	=	1978	5	0
	..... 1*s.* 8*d.*	=	659	8	4
	..... $2\frac{1}{2}$*d.*	=	82	8	$6\frac{1}{2}$

Exer.			£	*s.*	*d.*
35.	Parts, 10*s.*, 2*s.* 6*d.* (=¼ of 10*s.*), 1*s.*, 7½*d.* (=¼ of 2*s.* 6*d.*)				
	Price at 10*s.*	=	2132	10	0
	..... 2*s.* 6*d.*	=	533	2	6
	..... 1*s.*	=	213	5	0
	..... 7½*d.*	=	133	5	7½
36.	Parts, 10*s.*, 2*s.* 6*d.*, 1*s.* 3*d.*; or 10*s.*, 3*s.* 4*d.*, 5*d.*				
	Price at £5	=	1245	0	0
	..... 10*s.*	=	124	10	0
	..... 2*s.* 6*d.*	=	31	2	6
	..... 1*s.* 3*d.*	=	15	11	3
	..... 3*s.* 4*d.*	=	41	10	0
	..... 5*d.*	=	5	3	9
37.	Parts, 2*s.* 6*d.*, 15*s.* (=six times 2*s.* 6*d.*), 1*s.* 3*d.*; or 10*s.*, 5*s.*, 2*s.* 6*d.*, 1*s.* 3*d.*				
	Price at 2*s.* 6*d.*	=	72	0	0
	..... 15*s.*	=	432	0	0
	..... 1*s.* 3*d.*	=	36	0	0
	..... 10*s.*	=	288	0	0
	..... 5*s.*	=	144	0	0
38.	Parts, 10*s.*, 6*d.* (=$\frac{1}{20}$ of 10*s.*); or 2*s.* 6*d.*, 7*s.* 6*d.* (=2*s.* 6*d.* × 3), 6*d.* (=⅕ of 2*s.* 6*d.*)				
	Price at £2	=12970	0	0	
	..... 10*s.*	=	3242	10	0
	..... 6*d.*	=	162	2	6
	..... 2*s.* 6*d.*	=	810	12	6
	..... 7*s.* 6*d.*	=	2431	17	6

Otherwise:—Since £2 - 10 - 6=50*s.* 6*d.*, to find the price in shillings, multiply by 50, and to the product add one half of 6485; or, since four times £2 - 10 - 6 is £10 - 2, find the price at £10 - 2, and divide it by 4.

40.	Price at 2*s.*	=	41	16	0
	..... 1*s.*	=	20	18	0
42.	..... 6*s.*	=	113	8	0
	..... 1*s.*	=	18	18	0
44.	..... 8*s.*	=	374	16	0
	..... 1*s.*	=	46	17	0
46.	..... 12*s.*	=	405	0	0
	..... 1*s.*	=	33	15	0

Exer.		£	*s.*	*d.*
49.	Price at 16*s.* ..... =	694	8	0
	..... 1*s.* ..... =	43	8	0
51.	..... £3 ..... =	2949	0	0
	..... 4*s.* ..... =	196	12	0
52.	..... £5 ..... =	3620	0	0
	..... 12*s.* ..... =	434	8	0
	..... 1*s.* ..... =	36	4	0
53.	..... 6*s.* 8*d.* ..... =	119	6	8
54.	..... 1½*d.* ..... =		74	10½
55.	..... 4*d.* ..... =		322	4
56.	..... £3 ..... =	825	0	0
	..... 5*s.* ..... =	68	15	0
57.	..... £4 ..... =	1444	0	0
	..... 8*s.* ..... =	144	8	0
58.	..... 3*d.* ..... =		222	3
59.	..... £6 ..... =	2898	0	0
	..... 3*s.* 4*d.* ..... =	80	10	0
60.	..... 5*s.* ..... =	47	5	0
	..... 5*d.* ..... =	3	18	9
61.	..... 1*d.* ..... =		83	1
62.	..... 2*s.* ..... =	75	6	0
	..... 3*d.* ..... =	9	8	3
63.	..... 1*s.* 8*d.* ..... =	54	1	8
	..... 2½*d.* (=⅛ of 1*s.* 8*d.*) =	6	15	2½
64.	..... 1*s.* ..... =	36	1	0
65.	..... £1 ..... =	328	15	0
	..... 4*s.* ..... =	65	15	0
	..... 2*s.* 6*d.* ..... =	41	1	10½
66.	..... £1 ..... =	675	10	0
	..... £2 ..... =	1351	0	0
	..... 10*s.* ..... =	337	15	0
	..... 4*s.* ..... =	135	2	0
	..... 4*d.* ..... =	11	5	2
67.	..... £1 ..... =	7538	15	0
	..... 2*s.* ..... =	753	17	6
	..... 4*d.* ..... =	125	12	11
68.	..... £1 ..... =	176	15	0
	..... 10*s.* ..... =	88	7	6
	..... 6*s.* 8*d.* ..... =	58	18	4
	..... 2*s.* ..... =	17	13	6
69.	..... £1 ..... =	164	17	6
	..... £2 ..... =	329	15	0

Exer.			£	*s.*	*d.*
	Price at 5*s.*	=	41	4	$4\frac{1}{2}$
	..... 6*d.*	=	4	2	$5\frac{1}{4}$
70.	..... £1	=	239	15	0
	..... 10*s.*	=	119	17	6
	..... 10*d.* (=$\frac{1}{12}$ of 10*s.*)	=	9	19	$9\frac{1}{2}$
71.	..... £1	=	172	14	0
	..... £3	=	518	2	0
	..... 10*s.*	=	86	7	0
	..... 5*s.*	=	43	3	6
	..... 10*d.*	=	7	3	11
72.	..... £1	=	257	18	4
	..... £2	=	515	16	8
	..... 5*s.*	=	64	9	7
	..... 4*s.*	=	51	11	8
	..... 4*d.*	=	4	5	$11\frac{2}{3}$
73.	Parts, 10*s.*, 4*s.* 6*d.*; or 10*s.*, 2*s.* 6*d.*, 2*s.*				
	Price at £1	=	75	18	9
	..... £4	=	303	15	0
	..... 10*s.*	=	37	19	$4\frac{1}{2}$
	..... 4*s.*	=	15	3	9
	..... 6*d.*	=	1	17	$11\frac{1}{2}$
	..... 2*s.* 6*d.* (=$\frac{1}{4}$ of 10*s.*)	=	9	9	10
	..... 2*s.*	=	7	11	$10\frac{1}{2}$
74.	..... £1	=	285	16	3
	..... £3	=	857	8	9
	..... 10*s.*	=	142	18	$1\frac{1}{2}$
	..... 6*s.* 8*d.*	=	95	5	5
	..... 1*s.*	=	14	5	$9\frac{3}{4}$
75.	Parts, 10*s.*, 4*s.*, 3*s.* 4*d.*; or 10*s.*, 6*s.* 8*d.*, 8*d.* (=$\frac{1}{10}$ of 6*s.* 8*d.*)				
	Price at £1	=	117	6	3
	..... 10*s.*	=	58	13	$1\frac{1}{2}$
	..... 4*s.*	=	23	9	3
	..... 3*s.* 4*d.* (=$\frac{1}{3}$ of 10*s.*)	=	19	11	$0\frac{1}{2}$
	..... 6*s.* 8*d.*	=	39	2	1
	..... 8*d.* (=$\frac{1}{10}$ of 6*s.* 8*d.*)	=	3	18	$2\frac{1}{2}$
76.	..... £1	=	84	17	6
	..... £12	=	1018	10	0
	..... 10*s.*	=	42	8	9
	..... 1*s.* 8*d.* (=$\frac{1}{6}$ of 10*s.*)	=	7	1	$5\frac{1}{2}$
	..... 6*s.* 8*d.*	=	28	5	10
	..... 5*s.*	=	21	4	$4\frac{1}{2}$

Exer.		£	*s.*	*d.*
77.	Price at £1 ..... =	134	8	9
	..... 1*s.* 8*d.* ..... =	11	4	$0\frac{3}{4}$
78.	..... £1 ..... =	836	13	9
	..... £2 ..... =	1673	7	6
	..... 5*s.* ..... =	209	3	$5\frac{1}{4}$
	..... 3*s.* 4*d.* ..... =	139	8	$11\frac{1}{2}$
79.	..... £1 ..... =	812	16	3
	..... £6 ..... =	4876	17	6
	..... 10*s.* ..... =	406	8	$1\frac{1}{2}$
	..... 2*s.* ..... =	81	5	$7\frac{3}{4}$
	..... 8*d.* (=$\frac{1}{3}$ of 2*s.*)..... =	27	1	$10\frac{1}{2}$
80.	..... £1 ..... =	176	7	6
	..... £2 ..... =	352	15	0
	..... 10*s.* ..... =	88	3	9
	..... 5*s.* ..... =	44	1	$10\frac{1}{2}$
	..... 1*s.* ..... =	8	16	$4\frac{1}{2}$
	..... 10*d.* ..... =	7	6	$11\frac{3}{4}$
	..... 1*s.* 8*d.* ..... =	14	13	$11\frac{1}{2}$
	..... 2*d.* ..... =	1	9	$4\frac{3}{4}$
81.	..... £1 ..... =	75	7	10·29
	..... 5*s.* ..... =	18	16	11·57
	..... 4*s.* ..... =	15	1	6·86
	..... 6*d.* ..... =	1	17	8·36
	..... 3*d.* ..... =	0	18	10·18
82.	Parts, 6*s.* 8*d.*, 3*s.* 4*d.*, 4*d.*; or 2*s.*, 8*s.*, 4*d.*			
	Price at £1 ..... =	538	13	0·43
	..... 6*s.* 8*d.* ..... =	179	11	0·14
	..... 3*s.* 4*d.* ..... =	89	15	6·07
	..... 4*d.* ..... =	8	19	6·61
	..... 2*s.* ..... =	53	17	3·64
	..... 8*s.* (=2*s.* × 4) ..... =	215	9	2·56
83.	..... £1 ..... =	346	5	8·57
	..... 10*s.* ..... =	173	2	10·28
	..... 2*s.* ..... =	34	12	6·86
	..... 6*d.* ..... =	8	13	1·71
	..... 1*d.* ..... =	1	8	10·28
84.	Parts, 2*s.* 6*d.*, 15*s.*, 1*s.* 3*d.*; or 10*s.*, 5*s.*, 2*s.* 6*d.*, 1*s.* 3*d.*			
	Price at £1 ..... =	786	11	5·14
	..... 2*s.* 6*d.* ..... =	98	6	5·14
	..... 15*s.*(=6 times 2*s.* 6*d.*)=	589	18	6·84

Exer.		£	*s.*	*d.*
	Price at 1*s.* 3*d.* (=$\frac{1}{2}$ of 2*s.* 6*d.*) =	49	3	2·57
	..... 10*s.* ..... =	393	5	8·57
	..... 5*s.* ..... =	196	12	10·28
85.	..... £1 ..... =	647	11	11·57
	..... £6 ..... =	3885	11	9·42
	..... 6*s.* 8*d.* ..... =	215	17	3·86
	..... 4*s.* ..... =	129	10	4·71
86.	..... £1 ..... =	238	0	6·43
	..... £3 ..... =	714	1	7·29
	..... 10*s.* ..... =	119	0	3·21
	..... 5*s.* ..... =	59	10	1·60
	..... 4*s.* ..... =	47	12	1·29
	..... 7$\frac{1}{2}$*d.* (=$\frac{1}{8}$ of 5*s.*) ... =	7	8	9·20
87.	..... £1 ..... =	181	17	3·86
	..... £3 ..... =	545	11	11·58
	..... 6*s.* 8*d.* ..... =	60	12	5·29
	..... £2 ..... =	363	14	7·72
	..... 10*s.* ..... =	90	18	7·93
	..... 3*s.* 4*d.* ..... =	30	6	2·64
88.	Parts, 1*s.* 8*d.*, 15*s.*, 5*d.*; or 10*s.*, 5*s.*, 1*s.* 8*d.*, 5*d.*			
	Price at £1 ..... =	251	10	2·14
	..... 1*s.* 8*d.* ..... =	20	19	2·18
	..... 15*s.* ..... =	188	12	7·62
	..... 5*d.* (=$\frac{1}{4}$ of 1*s.* 8*d.*) =	5	4	9·54
	..... 10*s.* ..... =	125	15	1·07
	..... 5*s.* ..... =	62	17	6·53
	..... 1*s.* 8*d.* ..... =	20	19	2·18
89.	..... £1 ..... =	103	4	9·86
	..... £5 ..... =	516	4	1·30
	..... 10*s.* .. .. =	51	12	4·93
	..... 4*s.* ..... =	20	12	11·57
	..... 10*d.* (=$\frac{1}{12}$ of 10*s.*) =	4	6	0·41
90.	..... £1 ..... =	418	13	0·43
	..... £2 ..... =	837	6	0·86
	..... 8*d.* (=$\frac{1}{30}$ of £1) ... =	13	19	1·21
91.	Parts, 10*s.*, 1*s.* 3*d.* (=$\frac{1}{8}$ of 10*s.*)			
	Price at £1 ..... =	179	19	5·57
	..... £3 ..... =	539	18	4·71
	..... 10*s.* ..... =	89	19	8·78
	..... 1*s.* 3*d.* ..... =	11	4	11·60

Exer.		£	*s.*	*d.*
92.	Price at £1 ..... =	246	19	3·43
	..... £3 ..... =	740	17	10·29
	..... 3*s.* 4*d.* ..... =	41	3	2·57
	..... 2*s.* ..... =	24	13	11·14
	..... 4*s.* ..... =	49	7	10·29
	..... 1*s.* 4*d.* ..... =	16	9	3·43
93.	Parts, 2*s.* 6*d.*, 15*s.*, 1*s.*, 1½*d.*; or 10*s.*, 5*s.*, 2*s.* 6*d.*, 6*d.*, 7½*d.*; or 10*s.*, 5*s.*, 2*s.* 6*d.*, 1*s.*, 1½*d.*			
	Price at £1 .... =	319	6	7·29
	..... 2*s.* 6*d.* ..... =	39	18	3·91
	..... 15*s.* (2*s.* 6*d.* × 6) ... =	239	9	11·46
	..... 1*s.* ..... =	15	19	3·96
	..... 1½*d.* (=⅛ of 1*s.*) ... =	1	19	10·99
	..... 10*s.* ..... =	159	13	3·64
	..... 5*s.* ..... =	79	16	7·82
	..... 6*d.* ..... =	7	19	7·98
	..... 7½*d.* (=⅛ of 5*s.*) ... =	9	19	6·98
94.	..... £1 ..... =	90	11	9·43
	..... £5 ..... =	452	18	11·15
	..... 2*s.* ..... =	9	1	2·14
	..... 4*d.* ..... =	1	10	2·35
	..... ½*d.* ..... =	0	3	9·29
95.	..... £1 ..... =	561	12	6
	..... £2 ..... =	1123	5	0
	..... 2*s.* 6*d.* ..... =	70	4	0¾
	..... 15*s.* (=2*s.* 6*d.* × 6) =	421	4	4½
96.	..... £1 ..... =	45	14	4·50
	..... 10*s.* ..... =	22	17	2·25
	..... 4*s.* ..... =	9	2	10·50
	..... 2*s.* 6*d.* (=¼ of 10*s.*) =	5	14	3·56
97.	..... £1 ..... =	77	8	9
	..... 10*s.* ..... =	38	14	4·50
	..... 2*s.* ..... =	7	14	10·50
98.	..... £1 ..... =	586	8	10·50
	..... 5*s.* ..... =	146	12	2·62
	..... 1*s.* 8*d.* ..... =	48	17	4·87
	..... 7½*d.* ..... =	18	6	6·33
99.	..... £1 ..... =	674	13	7·50
	..... 10*s.* .... =	337	6	9·75
	..... 1*s.* 3*d.* (=⅛ of 10*s.*) =	42	3	4·22
	..... 1½*d.* (=1/10 of 1*s.* 3*d.*) =	4	4	4·02

Exer.		£	*s.*	*d.*
100.	Price at £1 ..... =	311	13	3
	..... 2*s.* 6*d.* ..... =	38	19	1·87
	..... 3*d.* ..... =	3	17	10·99
101.	..... £1 ..... =	1268	17	1·50
	..... 4*s.* ..... =	253	15	5·10
	..... 2*s.* 6*d.* ..... =	158	12	1·69
102.	..... £1 ..... =	139	19	10·50
	..... £2 ..... =	279	19	9
	..... 5*s.* ..... =	34	19	11·62
	..... 2*s.* ..... =	13	19	11·85
	..... 10*d.* ..... =	5	16	7·94
103.	..... £1 ..... =	175	18	3
	..... £6 ..... =	1055	9	6
	..... £36 ..... =	6332	17	0
	..... £2 ..... =	351	16	6
	..... £38 ..... =	6684	13	6
	..... 10*s.* ..... =	87	19	1·50
	..... 2*s.* ..... =	17	11	9·90
	..... 1*s.* ..... =	8	15	10·95
104.	..... £1 ..... =	219	16	9
	..... £11 ..... =	2418	4	3
	..... 5*s.* ..... =	54	19	2·25
	..... 2*s.* 6*d.* ..... =	27	9	7·12
105.	..... £1 ..... =	93	7	7·50
	..... 6*s.* 8*d.* ..... =	31	2	6·50
	..... 3*s.* 4*d.* ..... =	15	11	3·25
	..... 4*d.* ..... =	1	11	1·52
106.	..... £1 ..... =	263	16	4·50
	..... 10*s.* ..... =	131	18	2·25
	..... 1*s.* 3*d.* ..... =	16	9	9·28
107.	..... £1 ..... =	58	16	3
	..... 10*s.* ..... =	29	8	$1\frac{1}{2}$
	..... 2*s.* ..... =	5	17	$7\frac{1}{2}$
	..... 8*d.* ..... =	1	19	$2\frac{1}{2}$
108.	..... £1 ..... =	105	13	9
	..... £2 ..... =	211	7	6
	..... 5*s.* ..... =	26	8	$5\frac{1}{4}$
	..... 3*s.* 4*d.* ..... =	17	12	$3\frac{1}{2}$
109.	Price of 8 cwt..... =	14	6	0
	..... 2 qrs..... =	0	17	10·50
	..... 8 lbs. ..... =	0	2	6·64
	..... 4 lbs. ..... =	0	1	3·32

Exer.		£	s.	d.
110.	Price of 8 acres ..... =	7	10	8
	..... 2 roods ..... =	0	9	5
	..... 1 rood ..... =	0	4	8·50
	..... 10 perches..... =	0	1	2·12
	..... 5 perches ..... =	0	0	7·06
	..... 4 perches ..... =	0	0	5·65
111.	..... 9 tons..... =	53	17	0
	..... 10 cwt. ..... =	2	19	10
	..... 2 cwt..... =	0	11	11·60
	..... 1 cwt..... =	0	5	11·80
112.	..... 11 acres ..... =	12	19	10·50
	..... 1 rood ..... =	0	5	10·87
	..... 20 perches..... =	0	2	11·43
	..... 2 perches ..... =	0	0	3·54
	..... 1 perch ..... =	0	0	1·77
113.	..... 2 yards ..... =	2	18	6
	..... 2 nails(=$\frac{1}{8}$ of 1 yard) =	0	3	$7\frac{7}{8}$
	..... 2 qrs. ..... =	0	14	7·50
	..... 1 qr. ..... =	0	7	3·75

MISCELLANEOUS EXERCISES.

(Arithmetic, page 216.)

114.	Price at 6*s.* 8*d.* ..... =	119	13	4
	..... 10*d.* ..... =	14	19	2
	..... 1*d.* ..... =	1	9	11
115.	..... 5*s.* ..... =	177	10	0
	..... 2*s.* 6*d.* ..... =	88	15	0
	..... $2\frac{1}{2}$*d.* (=$\frac{1}{12}$ of 2*s.* 6*d.*) ... =	7	7	11
116.	..... 10*s.* ..... =	62	0	0
	..... 1*s.* 3*d.* ..... =	7	15	0
	..... $1\frac{1}{2}$*d.* ($\frac{1}{10}$ of 1*s.* 3*d.*) ..... =	0	15	6
117.	Duty on 112 lbs., at 5*s.* ..... =	28	0	0
	..... 2*s.* 6*d.* ..... =	14	0	0
	..... 1*s.* 3*d.* ..... =	7	0	0
118.	Duty on 47c.0q.19 lbs., at £1 per cwt. =	47	3	4·71
	..... £5 ..... =	235	16	11·55
	..... 6*s.* 8*d.* ... =	15	14	5·57
	Duty on 197 lbs., at 2*s.*6*d.*..... =	24	12	6
	..... 1*s.* 3*d.* ..... =	12	6	3
	..... $2\frac{1}{2}$*d.*($\frac{1}{5}$ of 1*s.*3*d.*)=	2	1	$0\frac{1}{2}$

Exer.		£	*s.*	*d.*
119.	517 lbs. = 4 cwt. 2 qrs. 13 lbs. Then,			
	Duty on 4 cwt. ..... =	20	10	8
	..... 2 qrs. ..... =	2	11	4
	..... 8 lbs. (= $\frac{1}{14}$ of 1 cwt.) ... =	0	7	4
	..... 4 lbs. ..... =	0	3	8
	..... 1 lb. ..... =	0	0	11
120.	Duty at £1 ..... =	179	12	1·71
	..... £2 ..... =	359	4	3·42
	..... 2*s.* 6*d.* ..... =	22	9	0·21
	..... 10*d.* ..... =	7	9	8·07
	..... 2½*d.* ..... =	1	17	5·02

121. In solving this question, we may multiply the given prices by the number of barrels, and find the values of the stones and pounds by means of aliquot parts. For the four given quantities, the parts may be as follows :—1.—10 st. (= $\frac{1}{2}$ bar.), 5 st., 2 st., 7 lbs. (= $\frac{1}{4}$ of 2 st.), 2 lbs. (= $\frac{1}{14}$ of 2 st.) ; 2.—8 st. (= $\frac{1}{2}$ bar.), 2 st. 1., 4 lbs. (= $\frac{1}{7}$ of 2 st.), 1 lb. ; 3.—7 st. (= $\frac{1}{2}$ bar), 2 st. (= $\frac{1}{7}$ bar.), 2 st. 2 lbs. (= $\frac{1}{14}$ of 2 st.) ; 4.—6 st. (= $\frac{1}{2}$ bar.), 4 st. (= $\frac{1}{3}$ bar.), 7 lbs. (= $\frac{1}{8}$ of 2 st.), 4 lbs. (= $\frac{1}{14}$ of 4 st.) Then,

		£	*s.*	*d.*
1.	Price of 13 bar. ..... =	17	17	6
	..... 10 st. ..... =	0	13	9
	..... 5 st. ..... =	0	6	10½
	..... 2 st. ..... =	0	2	9
	..... 7 lbs. ..... =	0	0	8¼
	..... 2 lbs. ..... =	0	0	2¼
2.	..... 17 bar. ..... =	12	6	6
	..... 8 st. ..... =	0	7	3
	..... 2 st. ..... =	0	1	9¾
	..... 1 st. ..... =	0	0	11
	..... 4 lbs. ..... =	0	0	3
	..... 1 lb. ..... =	0	0	0¾
3.	..... 15 bar. ..... =	8	0	0
	..... 7 st. ..... =	0	5	4
	..... 2 st. ..... =	0	1	6¼
	..... 2 lbs. ..... =	0	0	1¼
4.	..... 11 bar. ..... =	10	14	6
	..... 6 st. ..... =	0	9	9
	..... 4 st. ..... =	0	6	6

Exer.			£	s.	d.
	Price of 7 lbs.	=	0	0	$9\frac{3}{4}$
	..... 4 lbs.	=	0	0	$5\frac{1}{2}$
122.	1. Cost of 4 dozen	=	5	12	0
	..... 3 bot. (=$\frac{1}{4}$ doz.)	=	0	7	0
	..... 2 bot. (=$\frac{1}{6}$ doz.)	=	0	4	8
	2. ..... 6 bot. (=$\frac{1}{2}$ doz.)	=	1	19	0
	..... 1 bot.	=	0	6	6
123.	Rent of 2 roods (=$\frac{1}{2}$ acre)	=	3	13	6
	..... 10 per. (=$\frac{1}{8}$ of 2 roods)	=	0	9	$2\frac{1}{4}$
	..... 5 per.	=	0	4	$7\frac{1}{2}$
	..... 2 per	=	0	1	10
124.	1. Price of 2 qrs.	=	1	6	3
	..... 8 lbs. (=$\frac{1}{7}$ of 2 qrs.)	=	0	3	9
	..... 4 lbs.	=	0	1	$10\frac{1}{2}$
	..... 1 lb.	=	0	0	$5\frac{1}{2}$
	2. ..... 12 lbs.	=	0	9	6
	..... 8 oz. (=$\frac{1}{2}$ lb.)	=	0	0	$4\frac{3}{4}$
	..... 2 oz.	=	0	0	$1\frac{1}{4}$
125.	..... 10 cwt. (=$\frac{1}{2}$ of 1 ton)	=	2	12	6
	..... 5 cwt.	=	1	6	3
	..... 2 cwt.	=	0	10	6
	..... 2 qrs. (=$\frac{1}{4}$ of 2 cwt.)	=	0	2	$7\frac{1}{2}$
	..... 1 qr.	=	0	1	$3\frac{3}{4}$
126.	1. ..... 16 lbs.	=	0	9	0
	..... $\frac{1}{4}$ lb.	=	0	0	$3\frac{1}{4}$
	2. ..... 12 lbs.	=	0	6	3
	..... $\frac{1}{4}$ lb.	=	0	0	$1\frac{1}{2}$
	3. ..... 9 lbs.	=	1	1	0
	..... 4 oz. (=$\frac{1}{4}$ lb.)	=	0	0	7
	..... 2 oz.	=	0	0	$3\frac{1}{2}$
127.	Value of 725, at 4*s*.	=	2900*s*.		
	..... $1\frac{1}{2}$*d*.	=		90	$7\frac{1}{2}$
	..... 4*s*. $1\frac{1}{2}$*d*.	=	149	10	$7\frac{1}{2}$
	Value of 87, at 10*s*.	=	43	10	0
	..... 2*s*.	=	8	14	0
	..... 10*d*. (=$\frac{1}{12}$ of 10*s*.)	=	3	12	6
	..... 12*s*. 10*d*.	=	55	16	6
	Value of 417, at 3*s*. 4*d*.	=	69	10	0
	..... 5*d*.	=	8	13	9
	..... $\frac{1}{2}$*d*.	=	0	17	$4\frac{1}{2}$
	..... 3*s*. $9\frac{1}{2}$*d*.	=	79	1	$1\frac{1}{2}$
128.	Price of 4 cwt.	=	19	9	4

Exer.			£	*s.*	*d.*
	Price of 2 qrs.	=	2	8	8
	..... 1 qr.	=	1	4	4
	..... 14 lbs.	=	0	12	2
	..... 8 lbs.	=	0	6	$11\frac{1}{2}$
	Whole price	=	24	1	$5\frac{1}{2}$
	Price of 83, at 4*s.*	=	16	12	0
	..... 3*s.* 4*d.*	=	13	16	8
	Whole price	=	30	8	8
129.	138, at 10*s.*	=	69	0	0
	..... 6*s.* 8*d.*	=	46	0	0
	..... 16*s.* 8*d.*	=	115	0	0
	130, at 2*s.*	=	13	0	0
	..... 14*s.*	=	91	0	0
	..... 4*d.*	=	2	3	4
	..... 16*s.* 4*d.*	=	106	3	4
	110, at 6*s.* 8*d.*	=	36	13	4
	..... 1*d.*	=	0	9	2
	..... 26*s.* 9*d.*	=	147	2	6
	120, at 10*s.*	=	60	0	0
	..... 4*s.*	=	24	0	0
	..... 10*d.*	=	5	0	0
	..... 14*s.* 10*d.*	=	89	0	0
130.	156, at £5	=	780	0	0
	..... 5*s.*	=	39	0	0
	..... 4*s.*	=	31	4	0
	..... 8*d.* (=$\frac{1}{6}$ of 4*s.*)	=	5	4	0
	..... £5 - 9 - 8	=	855	8	0
	313, at £3	=	939	0	0
	..... 4*s.*	=	62	12	0
	..... 1*s.* 8*d.* (=$\frac{1}{12}$ of £1)	=	26	1	8
	..... £3 - 5 - 8	=	1027	13	8
	93, at £3	=	279	0	0
	..... 5*s.*	=	23	5	0
	..... 2*s.*	=	9	6	0
	..... 1*s.* 3*d.*	=	5	16	3
	..... £3 - 8 - 3	=	317	7	3
	64, at £3	=	192	0	0
	..... 2*s.* 6*d.*	=	8	0	0
	..... 1*s.*	=	3	4	0
	..... £3 - 3 - 6	=	203	4	0
131.	218, at 2*s.*	=	21	16	0
	..... 1*s.*	=	10	18	0

Exer.		£	s.	d.
	218, at 2*d.* ..... =	1	16	4
	..... 3*s.* 2*d.* ..... =	34	10	4
	173, at 1*s.* ..... =	8	13	0
	..... 3*d.* ..... =	2	3	3
	..... 1½*d.* ..... =	1	1	7½
	..... 1*s.* 4½*d.* ..... =	11	17	10½
	56½, at 1*s.* ..... =	2	16	6
	..... 2*d.* ..... =	0	9	5
	..... 1*s.* 2*d.* ..... =	3	5	11
	27¾ at 1*s.* ..... =	1	7	9
	6*d.* ..... =	0	13	10½
	4*d.* ..... =	0	9	3
	10*d.* ..... =	1	3	1½
	Sum of all = £50 - 17 - 3; from which take £32 - 12 - 6			
132.	24 st. (=8×3) at 8*s.* 10*d.* ..... =	10	12	0
	16 st. at 7*s.* 4*d.* ..... =	5	17	4
	86 lbs. at 4*s.* 6*d.* ..... =	19	7	0
	38 gal. at 6*s.* 10*d.* ..... =	12	19	8
	10 gal. at 13*s.* 6*d.* ..... =	6	15	0

COMPOUND PROPORTION.

(Arithmetic, page 221.)

Exer.

1. As 43 cwt. : 59 cwt.
   £2 - 16 : £2 - 4
   :: 19 miles; or,
   as 43 cwt. : 59 cwt.
   56*s.* : 44*s.*
   :: 19 miles : $20\frac{2}{6}\frac{1}{0}\frac{1}{2}$ miles.

In this, 4 will divide 56 and 44.

2. As 40 miles : 65 miles
   45*s.* : 75*s.*
   :: 13 cwt.;

Or, by dividing by 5 and 15,

   as 8 : 13
   3 : 5
   :: 13 cwt. : $35\frac{5}{24}$ cwt.

3. As 18 days : 5 days
   11 acres : 33 acres
   :: 12 horses;

Or, by dividing the third and fourth terms by 11, the first and fifth by 6, and two of the quotients by 3,

   as 1 : 5
   1 : 1
   :: 2 horses : 10 horses.

Exer.

4. As 250 miles : 400 miles
   10 hours : 12 hours } :: 9 days;

Or, by dividing the first and second terms by 50, and the third and fourth by 2,

as 5 : 8
   5 : 6 } :: 9 days : $17\frac{7}{25}$ days.

5. As 6 persons : 8 persons
   £42 : £100 } :: 16 weeks;

Or, by dividing the first, second, third, and fifth terms by 2,

as 3 : 4
   21 : 100 } :: 8 weeks : $50\frac{5}{6}\frac{0}{3}$ weeks.

6. As 20 men : 29 men
   13 hours : 12 hours
   32 acres : 40 acres } :: 5 days;

Or, by dividing the first and sixth terms by 20, and the fourth and fifth by 4,

as 1 : 29
   13 : 3
   8 : 2 } :: 5 days;

Or, by dividing the fifth and sixth terms by 2,

as 1 : 29
   13 : 3
   4 : 1 } :: 5 days : $8\frac{1}{5}\frac{3}{2}$ days.

7. As 312*s.* : 702*s.*
   24 days : 18 days } :: 16 labourers;

Or, by dividing the first, second, third, and fourth terms by 6,

as 52 : 117
   4 : 3 } :: 16 labourers;

Or, by dividing the first and third each by 4, and the fifth by 16; and also the first of these quotients and the second term by 13,

as 1 : 9
   1 : 3 } :: 1 labourer : 27 labourers.

8. As 36 yards : 120 yards
   7 qrs. : 5 qrs. } :: 504*s.* : 1200*s.* or £60.

9. As 108 days : 270 days
   20 : 21 } :: 16 guineas : 42 sovereigns.

10. As 26 a. 2 r. 23 p. : 17 a. 3 r. 2 p.
    6 : 7 } :: £50 - 8 - 9;

Exer.

Or, by reduction,

as 4263 per. : 2842 per.
6 : 7
:: £50 - 8 - 9;

Or, by dividing the first and second terms by 1421, and the third term and the second quotient by 2,

as 3 : 1
3 : 7
:: £50 - 8 - 9 : £39 - 4 - 7.

11. As 85 galls. : 63 galls.
5 : 4
:: £58 - 8 - 9;

Or, by reduction,

as 85 : 63
5 : 4
:: 14025*d.* : 8316*d.* or £34 - 13.

12. As 23 men : 40 women
9 : 7
:: 6 days : $8\frac{8}{69}$ days.

13. 500 miles—191 miles=309 miles. Then,

as 191 miles : 309 miles
10 hours : 13 hours
:: 7 days : $14\frac{137}{191}$ days.

14. As 63 lbs. : 70 lbs.
10 lbs. : 9 lbs.
:: £20-10-6 : £20-10-6.

This solution, and some of the foregoing, may be contracted.

SIMPLE INTEREST.

(Arithmetic, page 227.)

Exer. 1.

£	s.	d.	
774	11	3,	at 5
		5	
3872	15	3	
38	14	$6\frac{3}{4}$	

Exer. 2.

£	s.	d.	
15	0	0,	at $4\frac{1}{2}$
		$4\frac{1}{2}$	
60	0	0	
7	10	0	
67	10	0	
0	13	6	

Exer. 3.

£	s.	d.	
39	12	6,	at $5\frac{1}{2}$
		$5\frac{1}{2}$	
198	2	6	
19	16	3	
217	18	9	
2	3	7	

Exer. 4.

£	s.	d.	
468	16	8,	at $3\frac{1}{2}$
		$3\frac{1}{2}$	
1406	10	0	
234	8	4	
1640	18	4	
16	8	$2\frac{1}{2}$	

Exer. 5.

£	s.	d.	
57	10	0,	at $4\frac{1}{2}$
		$4\frac{1}{2}$	
230	0	0	
28	15	0	
258	15	0	
2	11	9	

Exer. £ s. d.

6. 876 12 6, at $5\frac{3}{4}$
6
5259 15 0 for 6
219 3 $1\frac{1}{2}$ for $\frac{1}{4}$
100)5040 11 $10\frac{1}{2}$
50 8 $1\frac{1}{2}$

7. 9 13 4, at $3\frac{3}{4}$
$3\frac{3}{4}$
29 0 0
7 5 0
100)36 5 0
0 7 3

8. 376 12 8, at 4
4
100)1506 10 8
15 1 $3\frac{3}{4}$

9. 86 5 0, at $7\frac{7}{8}$
$7\frac{7}{8}$
603 15 0
75 9 $4\frac{1}{2}$
100)679 4 $4\frac{1}{2}$
6 15 10

10. 637 11 0, at $5\frac{5}{6}$
$5\frac{5}{6}$
3187 15 0
531 5 10
100)3719 0 10
37 3 $9\frac{3}{4}$

11. 17 7 0, at $3\frac{1}{2}$
$3\frac{1}{2}$
52 1 0
8 13 6
100)60 14 6
0 12 $1\frac{3}{4}$

Exer. £ s. d.

12. 899 10 0, at $4\frac{3}{4}$
5
4497 10 0
$\frac{1}{4}$... 224 17 6
100)4272 12 6
42 14 $6\frac{1}{4}$

13. 37 6 0, at $4\frac{1}{2}$
$4\frac{1}{2}$
149 4 0
18 13 0
100)167 17 0
1 13 $6\frac{3}{4}$

14. 534 4 0, at $7\frac{1}{4}$
$7\frac{1}{4}$
3739 8 0
133 11 0
100)3872 19 0
38 14 7

15. 671 19 6, at $4\frac{1}{4}$
$4\frac{1}{4}$
2687 18 0
167 19 $10\frac{1}{2}$
100)2855 17 $10\frac{1}{2}$
28 11 2

16. 10 10 0, at 6
6
100)63 0 0
0 12 $7\frac{1}{2}$

Exer.	£	s.	d.	
17.	81	10	0,	at $4\frac{3}{4}$
			5	
	407	10	0	
$\frac{1}{4}$...	20	7	6	
	387	2	6	
			2	y. 5 m.
	774	5	0	
4 m. = $\frac{1}{3}$	129	0	10	
1 m. = $\frac{1}{4}$	32	5	$2\frac{1}{2}$	
100)	935	11	$0\frac{1}{2}$	
	9	7	$1\frac{1}{4}$	

18.	24	0	0,	at $5\frac{1}{4}$
			$5\frac{1}{4}$	
	120	0	0	
	6	0	0	
	126	0	0	
4 m. = $\frac{1}{3}$	42	0	0	
100)	168	0	0	
	1	13	$7\frac{1}{4}$	

19.	419	7	9,	at $4\frac{3}{8}$
			$4\frac{3}{8}$	
	1677	11	0	
$\frac{2}{8}$ = $\frac{1}{4}$...	104	16	$11\frac{1}{4}$	
$\frac{1}{8}$ = $\frac{1}{2}$...	52	8	$5\frac{1}{2}$	
	1834	16	$4\frac{3}{4}$	
6 m. = $\frac{1}{2}$	917	8	$2\frac{1}{4}$	
4 m. = $\frac{1}{3}$	611	12	$1\frac{1}{3}$	
100)	3363	16	$8\frac{1}{2}$	
	33	12	$9\frac{1}{4}$	

20.	493	16	8,	at 6
			6	
	2963	0	0	
4 m. = $\frac{1}{3}$	987	13	4	
4 m. ...	987	13	4	
100)	4938	6	8	
	49	7	8	

Exer.	£	s.	d.	
21.	24	18	9,	at 6
			6	
	149	12	6	
2 m. = $\frac{1}{6}$	24	18	9	
100)	124	13	9	
	1	4	$11\frac{1}{4}$	

22.	427	8	8,	at $5\frac{3}{4}$
			6	
	2564	12	0	
$\frac{1}{4}$ ...	106	17	2	
	2457	14	10	
4 m. = $\frac{1}{3}$	819	4	$11\frac{1}{4}$	
1 m. = $\frac{1}{4}$	204	16	$2\frac{3}{4}$	
100)	3481	16	0	
	34	16	$4\frac{1}{4}$	

23.	92	12	0,	at $6\frac{1}{2}$
			$6\frac{1}{2}$	
	555	12	0	
	46	6	0	
	601	18	0	
6 m. = $\frac{1}{2}$	300	19	0	
4 m. = $\frac{1}{3}$	200	12	8	
100)	1103	9	8	
	11	0	$8\frac{1}{4}$	

24.	25	0	0,	at 5
			5	
	125	0	0	
6 m. = $\frac{1}{2}$	62	10	0	
3 m. = $\frac{1}{4}$	31	5	0	
110)	218	15	0	
	2	3	9	

Exer.	£	s.	d.	
25.	651	0	0,	at $4\frac{1}{2}$
			$4\frac{1}{2}$	
	2604	0	0	
	325	10	0	
	2929	10	0	
6m.=$\frac{1}{2}$	1464	15	0	
1m.=$\frac{1}{6}$	244	2	6	
100)	1708	17	6	
	17	1	$9\frac{1}{4}$	

26.	584	18	8,	at $3\frac{1}{8}$
			$3\frac{1}{8}$	
	1754	16	0	
	73	2	4	
	1827	18	4	
6m.=$\frac{1}{2}$	913	19	2	
3m.=$\frac{1}{2}$	456	19	7	
100)	3198	17	1	
	31	19	$9\frac{1}{4}$	

Exer.	£	s.	d.	
27.	4	7	6,	at 6
			6	
	26	5	0	
4 m.=$\frac{1}{3}$	8	15	0	
1 m.=$\frac{1}{4}$	2	3	9	
100)	10	18	9	
	0	2	$2\frac{1}{4}$	

28.	52	10	0,	at 6
			6	
	315	0	0	
2 m.=$\frac{1}{6}$	52	10	0	
100)	367	10	0	
	3	13	6	

(Arithmetic, page 229.)

29. Interest for a year=£22 - 16 - 6. Then, as 365 days : 31 days :: £22 - 16 - 6; or, as 365 : 31 :: 5478*d.* : 465$\frac{1}{4}$*d.*, or £1 - 18 - 9$\frac{1}{4}$*d.*

30. Interest for a year = £60; and £60 ÷ 365 = 0 - 3 - 3$\frac{3}{73}$*d.*, the interest for one day.

31. Interest for a year=£35. Then, as 365 days : 89 days :: £35 : £8 - 10 - 8$\frac{1}{4}$.

32. Interest for one year=£22 - 17 - 3$\frac{1}{2}$. Then, as 365 days : 264 days :: £22 - 17 - 3$\frac{1}{2}$: £16 - 10 - 9, the interest for 26 days.

33. Interest for one year=£25 - 14 - 9$\frac{1}{4}$. Then, as 365 days : 171 * days :: £25 - 14 - 9$\frac{1}{4}$: £12 - 1 - 2, the answer.

* In this, and in the like questions that follow, both in interest and discount, the number of days will be found in the manner pointed out in the Arithmetic, *page* 72: and the master will be able to check this part of the pupil's work by means of the solutions here given, as the number of days in each exercise will be found mentioned in its solution.

Exer.

34. Interest for one year=£3 - 7 - $1\frac{1}{4}$. Then, as 365 days : 136 days :: £3 - 7 - $1\frac{1}{4}$: £1 - 5 - 0, the interest required.

35. Interest for one year=£28 - 9 - $8\frac{1}{2}$. Then, as 365 days : 141 days :: £2 - 11 - $9\frac{1}{4}$: £1 - 0 - 0, very nearly.

36. Interest for one year=£49 - 10 - $2\frac{3}{4}$. Then, as 365 days : 173 days : £49 - 10 - $2\frac{3}{4}$: £23 - 9 - 4, the answer.

37. Interest for one year=£2 - 8 - $7\frac{1}{4}$. Then, as 365 days : 130 days :: £2 - 8 - $7\frac{1}{4}$: £0 - 17 - $3\frac{3}{4}$, the interest required.

38. Here, the time is 1 year, 315 days; and the interest of £15 - 15 - 0 for a year is 18*s.* $10\frac{3}{4}$*d.* Then, as 365 : 315 :: 18*s.* $10\frac{3}{4}$*d.* : 16*s.* $3\frac{3}{4}$*d.* Lastly, by adding together the two results, 18*s.* $10\frac{3}{4}$*d.* and 16*s.* $3\frac{3}{4}$*d.*, we get £1 - 15 - $2\frac{1}{2}$.

39. Here, the time is 2 years, 100 days; and the interest for 1 year is £2 - 8 - 0, and consequently, for 2 years, £4 - 16 - 0. Then, as 365 : 100 :: £2 - 8 - 0 : 13*s.* $1\frac{1}{4}$*d.* Lastly, £4 - 16 - 0 + 13*s.* $1\frac{3}{4}$*d.* = £5 - 9 - $1\frac{3}{4}$, the whole interest.

(Arithmetic, page 230.)

40.

£	s.	d.	
648	15	6	
176 days			
3888			
4536			
648			
			88 for 10*s.*
			44 ... 5*s.*
			4 ... 6*d.*
114184			
			$\frac{1}{3}$ ... 38061
			$\frac{1}{10}$ ... 3806
			$\frac{1}{10}$ ... 380
15|6431			
£15	12	$10\frac{1}{4}$	
		$\frac{1}{4}$, correction.	
£15	12	10	

Otherwise: Divide £114184 by 7300

Exer.

41. 14 × 12 = 168, and 168 × 224 = 37632. Then,

	37632
$\frac{1}{3}$ ...	12544
$\frac{1}{10}$ ...	1254
$\frac{1}{10}$ ...	125
	51555
£0 10	$3\frac{3}{4}$

Or, divide 37632 by 73000.

Exer. £ *s.* *d.*

42. 688 18 4
168 days
5504
4128
688
84 for 10*s.*
42 ... 5*s.*
28 ... 3*s.* 4*d.*
115738
12, twice the rate.
1388856
$\frac{1}{3}$ ... 462952
$\frac{1}{10}$ ... 46295
$\frac{1}{10}$ ... 4629
19|02732
£19 0 6$\frac{1}{3}$
$\frac{1}{2}$, correction.
£19 0 6

43. 884 8 8
239 days
7956
2652
1768
80 for 6*s.* 8*d.*
24 ... 2*s.*
211380
$\frac{1}{3}$ ... 70460
$\frac{1}{10}$ ... 7046
$\frac{1}{10}$ ... 704
28|9590
£28 19 2$\frac{1}{2}$
$\frac{1}{4}$, correction.
£28 19 1$\frac{1}{2}$

Exer. £ *s.* *d.*

44. 4868 15 0
146 days
29208
19472
4868
73, for 10*s.*
36 ... 5*s.*
710837
13, twice the rate.
9240881
$\frac{1}{3}$ ... 3080294
$\frac{1}{10}$ ... 308029
$\frac{1}{10}$ ... 30803
126|60007
£126 12 0
3, correction.
£126 11 9

45. 107 days
£66 8*s.*
642
642
21, for 4*s.*
21, ...
7104
11, twice the rate.
78144
$\frac{1}{3}$ ... 26048
$\frac{1}{10}$ ... 2605
$\frac{1}{10}$ ... 260
1|07057
£1 1 5

Exer.

46. By multiplying £14 by 18 (=6×3), and dividing by 100, we get £2 - 10 - 4¾, the interest for three years. Then,

122 days
£14
1708
12
20496
⅓ ... 6832
1/10 ... 683
1/10 ... 68
28079
£0 5 7¼

Lastly, add the two results together.

47.

180 days
£70 6*s.*
12600
36, for 4*s.*
18, ... 2*s.*
12654
1/10 ... 1265
13919
12×4+2 ... 50
1|3869
£1 7 8¾

Exer.

48.

	£	*s.*	*d.*
	593	12	6
170 days			
	41510		
	593		
85, for 10*s.*			
21, ... 2*s.* 6*d.*			
	100916		
1/10 ...	10092		
	111008		
100×4+4	404		
	11|0604		
	£11	1	2½
Correction,			¼
	£11	1	2¼

49.

	374	5	0
272 days			
	748		
	2618		
	748		
68, for 5*s.*			
	101796		
1/10 ...	10180		
	11·1976		
101×4+3...	407		
	11|1569		
	£11	3	1¾
Correction,			¼
	£11	3	1½

Exer.

50. £247
86 days
1482
1976
21242, for 4 p. cnt.
10621, ... 2 .....
31863, ... 6 .....
$\frac{1}{10}$ ... 3186
35049
31×4+3 127
3|4922
£3 9 10

51. 139 days
£30 2*s.*
4170
14, for 2*s.*
4184, for 4 p. cnt.
2092, ... 2 .....
6276, ... 6 .....
$\frac{1}{10}$ ... 628
6904
6×4+1 25
6879
£0 13 9

Exer. £ *s.* *d.*

52. 176 11 4
161 days
176
1056
176
80, for 10*s.*
8, ... 1*s.*
3, ... 4*d.*
28427, for 4 p. cnt.
$\frac{1}{4}$ ... 7107, ... 1 .....
$\frac{1}{2}$ ... 3553, ... $\frac{1}{2}$ .....
39087, ... $5\frac{1}{2}$.....
$\frac{1}{10}$ ... 3909
42996
39×4... 156
4|2840
£4 5 8

(Arithmetic, page 233.)

Exer. 54.

	£	s.	d.	*Days.*		*Dr.*	*Cr.*
May 19, To	512	12	6	× 244	.....	125080	
Aug. 23, To	273	8	0	× 148	.....	40463	
Oct. 4, To	186	10	0	× 106	.....	19769	
Nov. 18, To	272	5	0	× 61	.....	16607	
June 13, By	400	18	0	× 219	.....		87797
Nov. 8, By	680	0	0	× 71	.....		48280
Dec. 1, By	73	5	8	× 48	.....		3518
						201919	139595
						139595	
						62324, for 4 p. c.	
					½ ...	31162, ... 2	
						93486, ... 6	
					$\frac{1}{10}$ ...	9349	
						102835	
				93×4+2 ...		374	
						10|2461	
						£10 4 11¼	
					Correction,	¼	
					Interest,	£10 4 11	

(Arithmetic, page 234.)

Exer. 55.

	£	s.	d.	*Days.*	*Dr*	*Cr.*
Sep. 3, To	1280	3	11	× 296	378938	
Dec. 21, To	793	18	0	× 187	148459	
Jan. 27, To	1040	5	0	× 150	156037	
Mar. 26, To	838	14	2	× 92	77161	
Sept. 19, By	1510	10	0	× 280		422940
Mar. 20, By	1248	8	0	× 98		122343
May 25, By	912	16	8	× 32		29211
					760595	574494
					574494	
					186101, for 4 p. c.	
				¼ ...	46525, ... 1	
					232626, ... 5	
				1/10 ...	23263	
					255889	
				232 × 4 + 2 ...	930	
					25|4959	
					£25 9 11	
				Correction,	½	
				Interest,	£25 9 10½	

(Arithmetic, page 234.)

Exer. 56.

Leap-year.	£	s.	d.	*Days.*	*Dr.*	*Cr.*
Feb. 9, To	768	8		9×263	202099	
May 7, To	436	17		6×175	76453	
July 8, To	948	5		10×113	107157	
Aug. 18, To	673	11		0× 72	48496	
April 1, By	419	4		8×211		88458
June 27, By	500	0		0×124		62000
Sept. 11, By	1533	12		6× 48		73614
					434205	224072
					224072	
					210133, for 4 p.c.	
				½ ...	105067, ... 2 ...	
					315200, ... 6 ...	
				⅒ ...	31520	
					346720	
				315×4+1 ...	1261	
					34|5459	
					£34 10 11	
				Correction,	1,	nearly.
					£34 10 10	

Exer.

57. As £6 : £100 } :: £24 : £634 - 15 - 7¾.
230 days : 365 days }

58. As £5 : £100 :: £341 - 5 : £6825.

59. In a leap-year, the time from January 1 to September 29, is 272 days. Then, 272×5½=1496, and 365×100=36500. Add these together; and then, as the sum, 37996 : 36500 :: £1000 : £296 - 12 - 6½.

60. 2 years, 10 months=34 months; and 34×4=136; also 12×100=1200. Then, 1200+136=1336; and as 1336 : 1200 :: £627 - 18 - 6 : £564 - 0 - 1.

61. £500−£460=£40, the interest. Then,
as £460 : £100 }
£4½ : £40 } :: 1 year : 1 year, 340$\frac{65}{207}$ days.

62. Here, £2280−£2000=£280, the interest. Then,
as £2000 : £100 }
£3½ : £280 } :: 1 year : 4 years.

Exer.

63. The amount of £780, at £1 - 2 - 9 is £887 - 5. Then,
as £887 - 10 : £100
£5¼ : £120
} :: 1 yr. : 2 yrs. $210\frac{20}{497}$ days.

64. As $2\frac{2}{4}$ years : 1 year
£5000 : £100
} :: £2000 : £14 - 10 - $10\frac{10}{11}$.

65. £1 - 2 - 9 − £1 = 2*s.* 9*d.*, the interest. Then,
As $3\frac{1}{4}$ years : 1 year
£1 : £100
} :: 2*s.* 9*d.* : £4 - 4 - $7\frac{5}{13}$.

66. 147043 − 100749 = 46294; 202426 − 147043 = 55383; and 282134 − 202426 = 79708. Then, as 100749 : 46294 :: 100 : 45·95, nearly; as 147043 : 55383 :: 100 : 37·664; and, as 202426 : 79708 :: 100 : 39·376.

67. 138235 − 102987 = 35248; 162403 − 138235 = 24168; and 164363 − 162403 = 1960. Then, as 102987 : 35248 :: 100 : 34·226, nearly; as 138235 : 24168 :: 100 : 17·483; and, as 162403 : 1960 :: 100 : 1·207, nearly.

68. 16262301 − 14391631 = 1870670; and 18526567 − 16262301 = 2264266. Then, as 14391631 : 1870670 :: 100 : 12·998; and, as 16262301 : 2264266 :: 100 : 13·923.

69. 7767401 − 6801827 = 965574; and 8205382 − 7767401 = 437981. Then, as 6801827 : 965574 :: 100 : 14·196, nearly; and, as 7767401 : 437981 :: 100 : 5·639, nearly.

DISCOUNT.

(Arithmetic, page 227.)

Exer. 1. This bill is due October 4th; and from June 9th till this date, there are 117 days. Then,

£416 3 4
117
2912
416
416
19, for 3*s.* 4*d.*
48691
$\frac{1}{10}$ ... 4869
53560
48×4+3 195
5|3365
£5 6 8$\frac{3}{4}$, discount.

Take this from the given amount, and the remainder will be the answer. The same is to be done in all the remaining questions, except the 16th and 17th.

Exer. 2. Here, the days from Nov. 12th till Feb. 8th, are 88. Hence,

£56
88
448
448
4928
9, twice the rate.
44352
$\frac{1}{3}$... 14784
$\frac{1}{10}$... 1478
$\frac{1}{10}$... 148
60762
£0 12 1$\frac{3}{4}$, discount.

Exer. 3. In this exercise, the bill is due on the 17th of December; and between the 3rd of October and this date, there are 75 days. Then,

£218 11 8
75
1090
1526
37, for 10*s.*
6, ... 1*s.* 8*d.*
16393
$\frac{1}{10}$ ... 1639
18032
16×4+1 65
1|7967
£1 15 11$\frac{1}{4}$, discnt.

Exer. 4. From July 10th till Oct. 25th, there are 107 days. Then,

£607 3 4
107
4249
607
18, for 3*s.* 4*d.*
64967
11, twice the rate.
714637
$\frac{1}{3}$... 238212
$\frac{1}{10}$... 23821
$\frac{1}{10}$... 2382
9|79052
£9 15 9$\frac{1}{2}$, discount.

Exer. 5. From May 9th till Dec. 8th, there are 213 days. Then,

£895 12 0
213
2685
895
1790
106, for 10*s.*
21, ... 2*s.*
190762
$\frac{1}{3}$ ... 63587
$\frac{1}{10}$ ... 6359
$\frac{1}{10}$ ... 636
26|1344
£26 2 8$\frac{3}{4}$
$\frac{3}{4}$, correction
£26 2 7$\frac{1}{2}$, discount.

Exer. 6. Between Nov. 30th and Feb. 28th, there are 90 days. Then,

£16 10 0
90
1440
45, for 10*s.*
1485
12$\frac{1}{2}$
17820
742
18562
$\frac{1}{3}$ ... 6187
$\frac{1}{10}$ ... 619
$\frac{1}{10}$ ... 62
·25430
£0 5 1, discount.

Exer. 7. From June 11th till Sept. 9th, there are 90 days. Then,

£588 12 8
90
52920
45, for 10*s.*
9, ... 2*s.*
3, ... 8*d.*
52977, for 4 p. c.
$\frac{1}{2}$ ... 26488, ... 2 .....
79465, ... 6 .....
$\frac{1}{10}$ ... 7946
87411
79×4+2 318
8|7093
£8 14 2$\frac{1}{4}$
$\frac{1}{4}$, correctn.
£8 14 2, discount.

Exer. 8. Between June 19th and Jan. 28th, there are 223 days. Then,

£486 18 8
223
1458
972
972
111, for 10*s.*
74, ... 6*s.* 8*d.*
22, ... 2*s.*
108585, for 4 p. c.
$\frac{1}{4}$ ... 27146, ... 1 .....
135731, ... 5 .....
$\frac{1}{10}$ ... 13573
149304
135×4+3 543
14|8761
£14 17 6$\frac{1}{4}$
$\frac{1}{4}$, correctn.
£14 17 6, discount.

Exer. 9. From June 4th till Sept. 28th, there are 116 days. Then,

£875 5 8
116
5250
875
875
23, for 4*s.*
10, ... 1*s.* 8*d.*
101533
$\frac{1}{3}$ ... 33844
$\frac{1}{10}$ ... 3384
$\frac{1}{10}$ ... 338
13|9099
£13 18 $2\frac{1}{4}$
$\frac{1}{4}$, correction.
£13 18 2, discount.

Exer. 10. From March 25th till June 11th, there are 78 days. Then,

£388 2 6
78
3104
2716
10, for 2*s.* 6*d.*
30274, for 4 p. c.
15137, ... 2 .....
45411, ... 6 .....
$\frac{1}{10}$ ... 4541
49952
45×4+2 182
4|9770
£4 19 $6\frac{1}{2}$, discount, nearly; the correction being about half a farthing.

Exer. 11. Between Sept. 12th and Jan. 19th, there are 129 days. Then,

£1000
129
129000
11, twice the rate.
1419000
$\frac{1}{3}$ ...473000
$\frac{1}{10}$... 47300
$\frac{1}{10}$... 4730
19|44030
£19 8 $9\frac{3}{4}$
$\frac{1}{2}$, correction.
£19 8 $9\frac{1}{4}$, discount.

Exer. 12. From June 3rd till Nov. 30th, there are 180 days. Then,

£568 12 9
180
45440
568
90, for 10*s.*
22, ... 2*s.* 6*d.*
2, ... 3*d.*
102354
$\frac{1}{3}$ ... 34118
$\frac{1}{10}$ ... 3412
$\frac{1}{10}$ ... 341
14|0225
£14 0 $5\frac{1}{4}$
$\frac{1}{4}$, correction.
£14 0 5, discount.

Exer.
13. £447 12 6
171
447
3129
447
85, for 10*s.*
21, ... 2*s.* 6*d.*
76543
11½, twice the rate
841973
38271
880244
⅓ ...293415
1/10... 29341
1/10... 2934
12|05934
£12 1 2¼
¼, correction.
£12 1 2, discount.

Exer. 14. Between May 8th and November 3rd, there are 179 days. Then,
179
£22 10*s.*
358
358
89, for 10*s.*
4027
13, twice the rate.
52351
⅓ ... 17450
1/10... 1745
1/10... 174
71720
£0 14 4, discount.

Exer. 15. From April 19th till August 12th, there are 115 days. Then,
£649 13 4
115
3245
649
649
57, for 10*s.*
19, ... 3*s.* 4*d.*
74711
11, twice the rate.
821821
⅓ ...273940
1/10... 27394
1/10... 2739
11|25894
£11 5 2½
¼, correction.
£11 5 2, discount.

Exer. 16. £24 - 16 × 5½ = £136 - 8; which divide by 100.

Exer. 17.
£549
32
1098
1647
17568
⅓ ... 5856
1/10 ... 586
1/10 ... 59
2|4069
£2 8 1½

18. £970 18 4
5
4854 11 8, for 5 p.c.
242 14 7,... ¼ ...
4611 17 1,... 4¾ ...
2305 18 6½.. 6 m.
384 6 5 ... 1 ...
100)7302 2 0½
£73 0 5, discount

Method of finding the TRUE ANSWERS *to the Exercises in Discount.*

Exer.

1. $117 \times 4 = 468$, and $36500 + 468 = 36968$. Then, as 36968 : 36500 :: £416 - 3 - 4 : £410 - 17 - $11\frac{1}{2}$.
2. $88 \times 4\frac{1}{2} = 396$, and $36500 + 396 = 36896$. Then, as 36896 : 36500 :: £56 : £55 - 7 - $11\frac{3}{4}$.
3. $75 \times 4 = 300$, and $36500 + 300 = 36800$. Then, as 36800 : 36500 :: £218 - 11 - 8 : £216 - 16 - $0\frac{1}{4}$.
4. $107 \times 5\frac{1}{2} = 588\frac{1}{2}$. Then, $36500 + 588\frac{1}{2} = 37088\frac{1}{2}$; and, as $37088\frac{1}{2}$: 36500 :: £607 - 3 - 4 : £597 - 10 - $7\frac{3}{4}$.
5. $213 \times 5 = 1065$. Then, $36500 + 1065 = 37565$; and, as 37565 : 36500 :: £895 - 12 : £870 - 4 - $2\frac{1}{4}$.
6. $90 \times 6\frac{1}{4} = 562\frac{1}{2}$, and $36500 + 562\frac{1}{2} = 37062\frac{1}{2}$. Then, as $37062\frac{1}{2}$: 36500 :: £16 - 10 : £16 - 5, nearly.
7. $90 \times 6 = 540$. Then, $36500 + 540 = 37040$; and, as 37040 : 36500 :: £588 - 12 - 8 : £580 - 1 - $0\frac{1}{2}$.
8. $223 \times 5 = 1115$, and $36500 + 1115 = 37615$. Then, as 37615 : 36500 :: £486 - 18 - 8 : £472 - 9 - $11\frac{3}{4}$.
9. $116 \times 5 = 580$. Then, $36500 + 580 = 37080$; and, as 37080 : 36500 :: £875 - 5 - 8 : £861 - 11 - $10\frac{1}{4}$.
10. $78 \times 6 = 468$. Then, $36500 + 468 = 36968$; and, as 36968 : 36500 :: £388 - 2 - 6 : £383 - 4 - $2\frac{3}{4}$.
11. $129 \times 5\frac{1}{2} = 709\frac{1}{2}$, and $36500 + 709\frac{1}{2} = 37209\frac{1}{2}$. Then, as $37209\frac{1}{2}$: 36500 :: £1000 : £980 - 18 - $7\frac{3}{4}$.
12. $180 \times 5 = 900$, and $36500 + 900 = 37400$. Then, as 37400 : 36500 :: £568 - 12 - 9 : £554 - 19 - 1.
13. $171 \times 5\frac{3}{4} = 983\frac{1}{4}$. Then, $36500 + 983\frac{1}{4} = 37483\frac{1}{4}$; and, as $37483\frac{1}{4}$: 36500 :: £447 - 12 - 6 : £435 - 17 - 8.
14. $179 \times 6\frac{1}{2} = 1163\frac{1}{2}$. Then, $36500 + 1163\frac{1}{2} = 37663\frac{1}{2}$; and, as $37663\frac{1}{2}$: 36500 :: £22 - 10 : £21 - 16 - $1\frac{1}{4}$.
15. $115 \times 5\frac{1}{2} = 632\frac{1}{2}$. Then, $36500 + 632\frac{1}{2} = 37132\frac{1}{2}$; and, as $37132\frac{1}{2}$: 36500 :: £649 - 13 - 4 : £638 - 12.
16. As $105\frac{1}{2}$: $5\frac{1}{2}$ :: £24 - 16 : £1 - 5 - $10\frac{1}{4}$.
17. $32 \times 5 = 160$, and $36500 + 160 = 36660$. Then, as 36660 : 160 :: £549 : £2 - 7 - 11.
18. $19 \times 4\frac{3}{4} = 90\frac{1}{4}$. Then, $1200 + 90\frac{1}{4} = 1290\frac{1}{4}$; and, as $1290\frac{1}{4}$: 1200 :: £970 - 18 - 4 : £903 - 0 - $0\frac{3}{4}$.

EXCHANGE.

(Arithmetic, page 244.)

Exer. 1. $943 \div 52\frac{7}{18} = 17{\cdot}98$, nearly.
2. $943 \div 15\frac{1}{2} = 60\frac{3}{4}d.$, nearly.
3. $943 \div 66 = 14{\cdot}29$.

CHAIN RULE.

(Arithmetic, page 249.)

Exer. 4.

Petersburg,	rubles	12000 = ?	English;
English,		£1 = 24 fr. 55 c.	Paris;
Paris,	francs	300 = 137½ fl.	Dutch;
Dutch,	florins	33 = 40 marks,	Hambro';
Hambro',	marks	100 = 89 fl. 70 cr.	Vienna;
Vienna,	creutzers	185 = 1 ruble,	Petersburg:

Or, by reduction, by multiplying the francs by 100, and the third term by 2 to remove the fraction—

Rubles	12000 = ?	English;
English	£1 = 2455	centimes;
Centimes	60,000 = 275	florins;
Florins	33 = 40	marks;
Marks	100 = 8970	creutzers;
Creutzers	185 = 1	ruble.

Hence, the value of the 12000 rubles, in English money, would be obtained by dividing the continual product of the numbers in the first column, by that of those in the second column. By merely indicating these operations, however, by means of the signs, the work may be contracted in the following manner: $\frac{12000 \times 60000 \times 33 \times 100 \times 185}{2455 \times 275 \times 40 \times 8970}$ $= \frac{6 \times 60000 \times 33 \times 100 \times 185}{491 \times 275 \times 897}$ (by dividing the first figure above by 10, 10, 4, and 5, and by dividing the fourth figure below by 10, the third by 10 and 4, and the first by 5) $= \frac{6 \times 60000 \times 20 \times 37}{491 \times 299}$ (by dividing the third figure above and below by 3, the second figure below by 5, 5, and 11, and the third figure above by 11, the fourth by 5, and the fifth by 5). Then, the continual product of the num-

bers in the upper line is 266400000, and the product of those in the lower line is 146809; and, dividing the former by the latter, we get £1814 - 12 - $0\frac{1}{2}$.

Again, the value of 12000 rubles, at 3*s*. $0\frac{1}{2}$*d*., is £1825; and the difference between this and the former result is £10 - 7 - $11\frac{1}{2}$, the answer.

PAR OF EXCHANGE.

(Arithmetic, p. 250.)

Exer. 5. British pence ? = 1000 rees;
6400 rees = 203 grains fine gold;
22 grains fine = 24 gr. British standard;
5760 grains B. S. = 1 lb. Troy;
40 lbs. Troy = £1869;
£1 = 240 pence:

$\frac{1000 \times 203 \times 24 \times 1 \times 1869 \times 240}{6400 \times 22 \times 5760 \times 40} = \frac{203 \times 24 \times 1869 \times 24}{64 \times 22 \times 576 \times 4}$ (by dividing first antecedent and consequent by 100, and by dividing first and last antecedents and third and last consequents by 10) $= \frac{203 \times 1869}{8 \times 11 \times 64}$ (by dividing the second antecedent and first consequent by 8, the second and fourth antecedents by 3 each, and the third consequent by 9, and the last antecedent by 2 and 4, and the second consequent by 2, and the last by 4) $= \frac{379497}{5632} = 67{\cdot}36$ pence per milree.

Exer. 6. Hambro' banco = £1,
5 shillings and 2 pence = 1 ounce B. S.
40 ounces B. S. = 37 ounces pure silver;
451 ounces pure silver = 60 marks Cologne;
1 mark Cologne pure silver = $27\frac{3}{4}$ Hambro' marks *banco*:

$\frac{240 \times 37 \times 60 \times 27\frac{3}{4}}{62 \times 40 \times 451} = \frac{3 \times 37 \times 15 \times 111}{31 \times 451}$ (by dividing the first antecedent and consequent by 2, the first antecedent and second consequent by 40, and by dividing the third, and multiplying the last antecedent by 4) $= \frac{184851}{13981} = 13$ marks $3\frac{1}{2}$ schillings, nearly.

COMPOUND FELLOWSHIP.

(Arithmetic, page 252.)

Exer. 1. Here, the respective products of the stocks and times are £1400 and £1600; the sum of which is £3000. Then,

As £3000 : £1400 :: £331 - 12 - 6 : £154 - 15 - 2, and
£3000 : £1600 :: £331 - 12 - 6 : £176 - 17 - 4.

2. Here, the products of the respective stocks and times are £1360 and £1680, the sum of which is £3040. Then,

As £3040 : £250 :: £1360 : £111 - 16 - 10¼, and
£3040 : £250 :: £1680 : £138 - 3 - 1¾

3. In this question, the products of the respective stocks and gains are £2486 - 5, £2010, and £3074 - 5. The sum of these is £7570 - 10. Then,

As £7570 - 10 : £439 - 18 - 8; or, by reduction,
1816920 : 105584 :: £2486 - 5 : £144 - 9 - 7¼;
1816920 : 105584 :: £2010 : £116 - 16 - 1;
1816920 : 105584 :: £3074 - 5 : £178 - 12 - 11¾.

4. Here, the products of the stocks and times are £3210, £2854 - 10, and £5362 - 10; and the sum of these is £11427. Then,

As £11427 : £370 :: £3210 : £103 - 18 - 9¼;
£11427 : £370 :: £2854 - 10 : £ 92 - 8 - 6½;
£11427 : £370 :: £5362 - 10 : £173 - 12 - 8¼.

5. Here, the several products are £5831, £2515 - 10, and £5404 - 13 - 4, the sum of which is £13751 - 3 - 4. Then,

As £13751 - 3 - 4 : £386 - 15; or, by multiplying both by 12,
£165014 : £4641 :: £5831 : £163 - 19 - 11;
£165014 : £4641 :: £2515 - 10 : £ 70 - 14 - 11½;
£165014 : £4641 :: £5404 - 13 - 4 : £152 - 0 - 1¼.

Exer.

6. Here, the products are £6344 - 5, £5478 - 13 - $1\frac{1}{2}$, and £5241; and their sum is £17063 - 18 - $1\frac{1}{2}$. Then,

As £17063 - 18 - $1\frac{1}{2}$: £568 - 15; or by reduction,
8190675 halfp. : 273000 halfp. :: £6344 - 5 - 0 : £211 - 9 - $1\frac{3}{4}$;
8190675 halfp. : 273000 halfp. :: £5478 - 13 - $1\frac{1}{2}$: £182 - 12 - $1\frac{3}{4}$;
8190675 halfp. : 273000 halfp. :: £5241 - 0 - 0 : £174 - 13 - $8\frac{1}{2}$.

7. Here, the products are £5193 - 8 - 9, £6232 - 2 - 6, and £8309 - 10; and their sum is £19735 - 1 - 3. Then,

As £19735 - 1 - 3 : £686 - 1 - 2; or, by reduction,
4736415*d.* : 164654*d.* :: £5193 - 8 - 9 : £180 - 10 - 10;
4736415*d.* : 164654*d.* :: £6232 - 2 - 6 : £216 - 13 - 0;
4736415*d.* : 164654*d.* :: £8309 - 10 - 0 : £288 - 17 - 4.

INVOLUTION.

(Arithmetic, page 255.)

Exer. 1. $678 \times 678 = 459684$.

2. $119 \times 119 = 14161$, the second power; and
$14161 \times 119 = 1685159$, the third power.

3. $75 \times 75 = 5625$, the second power; and
$5625 \times 5625 = 31640625$, the fourth power.

4. $86 \times 86 = 7396$, the second power;
$7396 \times 7396 = 54700816$, the fourth power; and
$54700816 \times 86 = 4704270176$, the fifth power.

The answer would have been found with equal facility, by multiplying the second power by 86 to find the third power, and the third power by the second, to find the fifth. The third power is 636056.

5. $9 \times 9 = 81$, the second power;
$81 \times 81 = 6561$, the fourth power;
$6561 \times 6561 = 43046721$, the eighth power; and
$43046721 \times 9 = 387420489$, the ninth power.

The following are the other powers of 9 below the ninth: Third power, 729; fifth power, 59049; sixth power, 531441; seventh power, 4782969.

Exer.

6. The sixth power of the numerator is 64, and the same power of the denominator, 729. Hence, the answer is $\frac{64}{729}$.

7. By reducing $2\frac{3}{4}$ to an improper fraction, we obtain $\frac{11}{4}$. Then, the fifth power of 11 is 161051, and the fifth power 4 is 1024. Hence, the answer is $\frac{161051}{1024}$; or, by reduction to a mixed number, $157\frac{283}{1024}$.

8. $3\frac{2}{7}$, by reduction to an improper fraction, becomes $\frac{23}{7}$; and, by raising both terms to their fourth powers, we get $\frac{279841}{2401}$; or, by reduction to a mixed number, $116\frac{325}{2401}$.

9. $4{\cdot}367 \times 4{\cdot}367 = 19{\cdot}070689$, the second power. Then, the rest of the work, for seven places of decimals, by the contracted method, will stand thus:—

19·07068900
986070·91
1907068900
1716362010
13349482
114424
15256
1716
363·6911788, or
363·691179, nearly

10. $1{\cdot}03 \times 1{\cdot}03 = 1{\cdot}0609$, the second power. Then,

1·0609000
9060·1
10609000
636540
9548
1·1255088, the fourth power.
8805521·1
11255088
1125509
225102
56275
5628
90
9
1·2667701, the eighth power.

1·2667701, the eighth power.
1077662·1

12667701
2533540
760062
76006
8867
887
1

1·6047064, the sixteenth power.
30·1

1·6047064
481412

1·6528476, or [power.
1·652848, nearly, the seventeenth

Exer.

11. 1·035 × 1·035 = 1·071225, the second power.
Then,

1·0712250
522170·1

10712250
749858
10712
2142
214
54

1·1475230, the fourth power.
325741·1

11475230
1147523
459009
80327
5738
229
34

1·3168090, the eighth power.

1·3168090, the eighth power.
908613·1

13168090
3950427
131681
79009
10534
119

1·7339860, the sixteenth power.
522170·1

17339860
1213790
17340
3468
347
87

1·8574892, or
1·857489, nearly, the eighteenth power.

Exer.

12. For the several powers of 1·04, the teacher may have recourse to Table I. at the end of the Arithmetic, looking for the index of the power in the first column, and for the power itself in the third column.

13. $1{\cdot}0475 \times 1{\cdot}0475 = 1{\cdot}09725625$, the second power. Then,

1·09725625
52652790·1

109725625
9875306
768079
21945
5486
658
22
5

1·20397126, the fourth power.

1·20397126, the fourth power.
62179302·1

120397126
24079425
361191
108357
8428
120
24
7

1·44954678, the eighth power.
97645944·1

144954678
57981871
5798187
1304592
72477
5798
870
101
13

2·10118587, the sixteenth power.
87645944·1

210118587
84047435
8404743
1891067
105059
8405
1261
147
17

3·04576721, the twenty-fourth power.

Exer.

14. For the several powers of 1·05, recourse may be had, as in Exercise 12, to the first and fourth columns of Table I.

The easiest method of working the question without the table, is to find successively the second, fourth, eighth, sixteenth, and thirty-second powers, and to divide the last by 1·05.

Exer.

15. $1{\cdot}055 \times 1{\cdot}055 = 1{\cdot}113025$, the second power. Then,

1·11302500
520311·1

111302500
11130250
1113025
333907
2226
557

1·23882465, the fourth power.
56428832·1

123882465
24776493
3716474
991060
99106
2478
496
74
6

1·53468652, the eighth power.
25686435·1

153468652
76734326
4604060
613875
92081
12277
921
77
3

2·35526272, the sixteenth power.

2·35526272, the sixteenth power.
25686435·1

235526272
117763136
7065788
942105
141316
18842
1413
118
5

3·61458995, the twenty-fourth power.
520311·1

361458995
36145899
3614590
1084377
7229
1807

4·02312897, or
4·0231290, nearly the twenty-sixth power.

Exer.

16. 1·07 × 1·07 = 1·1449, the second power; and 1·1449 × 1·1449 = 1·31079601, the fourth power. Then,

1·310796010
10697013·1

1310796010
393238803
13107960
917557
117972
7865
13

1·718186180, the eighth power.

1·718186180, the eighth power.
81681817·1

1718186180
1202730326
17181862
13745489
171819
137455
10309
172
137

2·952163749, the sixteenth power.
947361259·2

5904327498
2655947374
147608187
5904327
295216
177130
8856
2066
118
26

8·715270798, or [second power.
8·7152708, nearly, the thirty-

EXTRACTION OF THE SQUARE ROOT.

(Arithmetic, page 259.)

Exer.

1. 5 (2·236068
   4
42) 100
2 84
443) 1600
3 1329
4466) 27100
6 26796
4472) 3040
2683
357
358

2. ·50 (70710678, or ·7071068, nearly.
49
1407) 10000
7 9849
14141) 15100
1 14141
14142 9590
8485
1105
990
115
113
2

Exer.

3. ·70 (·8366600
64
163) 600
3 489
1666) 11100
6 9996
16726) 110400
6 100356
16732) 100440
100396
44

4. ·07) ·2645751
4
46) 300
6 276
524) 2400
4 2096
5285) 30400
5 26425
5290) 39750
37035
2715
2645
70
52

Exer.

5. ·06 { ·324494897, or
4 { ·32449490, nearly.

44) 200
4 176
484) 2400
4 1936
4889) 46400
9 44001
4898) 23990
19594
4396
3918
478
440
38
34
4

6. ·00'60 { ·0774597,
49 { nearly.

147) 1100
7 1029
1544) 7100
4 6176
1548) 9240
7742
1498
1393
105
108

Exer.

7. 7'85 (28·01785
4

48) 385
8 384
5601) 10000
1 5601
56021) 43990
39219
4771
4482
289
280
9

8. 78'·50 { 8·86002257,
64 { or 8·8600226,
168) 1450 { nearly.
8 1344
1766) 10600
6 10596
177200) 400000
354400
45600
35440
10160
8860
1300
1240
60

Exer.

9. 5′62 (23·70653918

```
      4
43 ) 162
      3   129
467 ) 3300
      7   3269
4740 6 ) 31000
          6   28436
4741 25 ) 255640
          5   237065
47413 0 ) 185775
              142391
               43384
               42671
                 713
                 474
                 239
                 237
                   2
```

10.

$3 \times 8 = 24$

```
      16
88 ) 800
      8   704
969 ) 9600
      9   8721
9788 ) 87900
      8   78304
97969 ) 959600
      9   881721
97978 ) 778790
          685851
           92939
           88180
            4759
            3919
             840
             784
              56
              59
```

4·898979486, nearly; and this is to be divided by 8 to find the answer.

Exer.

11. 13′·20 (3·633180425

```
      9
66 ) 420
      6   396
723 ) 2400
      3   2169
7263 ) 23100
      3   21789
72661 ) 131100
      1   72661
72662 ) 584390
          581302
            3088
            2906
             182
             145
              37
              36
               1
```

12. 17′28 (41·56921938

```
      16
81 ) 128
      1   81
825 ) 4700
      5   4125
8306 ) 57500
      6   49836
83129 ) 766400
      9   748161
83138 ) 182390
          166276
           16114
            8314
            7800
            7482
             318
             249
              69
              66
               3
```

Exer.

13. $80 \times 3 = 2'40$ (1 ; 25) 140 (5 ; 304) 1500 (4 ; 3089) 28400 (9 ; 30981) 59900 (1 ; 309829) 2891900 (9 ; 309838) 1034390 (3 ; 104875 ; 11924 ; 2629 ; 151 ; 155 — 15·491933385, nearly; and this is to be divided by 80 to find the answer.

Exer.

14. $1'37'50$ (1 ; 21) 37 (1 ; 227) 1650 (7 ; 2342) 6100 (2 ; 23446) 141600 (6 ; 23452) 9240 ; 7036 ; 2204 ; 2111 ; 93 ; 94 — 1·17260394, nearly.

This might also be wrought by reducing $1\frac{3}{8}$ to the improper fraction $\frac{11}{8}$, and dividing the square root of 8×11 by 8.

Exer.

15. 1·03'75 (1·018577439
1

201) 375
1 201

2028) 17400
8 16224

20365) 117600
5 101825

20370) 157750
142595

15155
14259

896
815

81
61

20
18

2

Exer.

16. 33) 5·7445626465
25

107) 800
7 749

1144) 5100
4 4576

11484) 52400
4 45936

114885) 646400
5 574425

1148906) 7197500
6 6893436

1148912) 3040640
2297824

742816
689347

53469
45956

7513
6893

620
574

46

Exer. 17. 3'33 (18·24828759

1

28) 233
8 224

362) 900
2 724

3644) 17600
4 14576

36488) 302400
8 291904

36496) 104960
72992

31968
29197

2771
2555

216
182

34
32

2

Exer. 18. 6'66 (25·80697575801

4

45) 266
5 225

508) 4100
8 4064

51606) 360000
6 309636

516129) 5036400
9 4645161

516138) 3912390
3612971

299419
258069

41350
41290

60
52

8

19. This exercise will be wrought by extracting the root of 5, as in Exercise 1, and dividing it by 3, the root of the denominator.

Exer.

20. $4 \times 11 = 44$ (6·63324958071

	36
126)	800
6	756
1323)	4400
3	3969
13263)	43100
3	39789
132662)	331100
2	265324
1326644)	6577600
4	5306576
1326648)	12710240
	11939840
	770400
	663324
	107076
	106132
	944
	928
	16
	13
	3

The root above found is to be divided by 11, to find the answer.

21. Here, $2 \times 3 = 6$; the root of which is found as in Exercise 5, except that the decimal point is placed after the first figure of the root. Then, to find the answer, the root is to be divided by 3.

22. By reducing the given number to an improper fraction, we obtain $\frac{49}{36}$. Then, by extracting successively the roots of the numerator and denominator, we get for answer $\frac{7}{6}$, or $1\frac{1}{6}$.

We might also find the answer in the following manner: $1\frac{13}{36} = 1{\cdot}361$; the square root of which is $1{\cdot}16$, or $1\frac{1}{6}$, as before.

Exer.

23. The given number is equivalent to $\frac{100}{9}$; the square root of which is $\frac{10}{3}$, or $3\frac{1}{3}$.

Or, the given number is equivalent to $11\cdot\dot{1}$; the square root of which is $3\cdot\dot{3}$, or $3\frac{1}{3}$, as before.

24. In extracting the root of 11 by the contracted method, the several remainders are 2, 11, 439, 4144, 16444, 317756; 524262, 59935, 236, and 37; and the root is 3·31662479035, which is to be divided by 4, the root of the denominator, to find the answer.

25. The given number is equivalent to the improper fraction $\frac{43}{7}$. Then, $43 \times 7 = 301$. In finding the root of this, the remainders are 2, 12, 171, 3244, 12199, 178951; 54578, 19879, 2530, and 101: and the root is 17·349351573, nearly; which is to be divided by 7 to find the answer.

The answer might also be found by extracting the root of $6\cdot\dot{1}42857\dot{1}$.

26. The given number is equivalent to 794·20; in extracting the root of which, the remainders are 3, 10, 459, 876, 31239, 305775; 239598, 14146, 2873, and 55.

27. Here, the remainders are 0, 2, 24, 246, 2468, 24579, 235679, 1345679; 123457, 12346, 1235, 124, and 13.

28. The remainders are 0, 26, 169, 4379, 60845, 427739, 5061584; 332698, 18428, 5857, 200, and 12.

29. The given number, by reducing the fraction to a decimal, and by division into periods, becomes 2′07′·69′44′44′, &c.: and the remainders are 1, 11, 33, 463, 17523, 22988, 2298844; 2812199, 218111, 16349, 1937, 208, and 6.

30. The remainders are 1, 126, 1, 100, 10000, 1000000, 9299975; 2299700, 485699, 122899, 14059, 1361, and 91.

31. Here, by the necessary preparations, the number whose root is to be found, is 3′49′67′·18′18′18′, &c.: and the remainders are 2, 25, 371, 3557, 19217, 51793, 1439417; 3174465, 182543, 32947, 3028, and 36.

EXTRACTION OF THE CUBE ROOT.

(Arithmetic, page 263.)

Exer.

1.

0	0	123 (4·9731899, or
4	16	64 4·97319, nearly.
4	16	59000
4	32	53649
8	4800	5351000
4	1161	5114473
120	5961	236527
9	1242	222441
129	720300	14086
9	10339	7419
138	730639	6667
9	10388	5935
1470	741027	732
7	45	668
1477	74147	64
7	45	
1484	74192	
7		
1491		

2.

0	0	517 (8·0259574
8	64	512
8	64	5000000
8	128	3849608
16	1920000	1150392
8	4804	965405
2400	1924804	184987
2	4808	173898
2402	1929612	11089
2	120	9662
2404	193081	1427
2	120	1352
2406	193201	75
	2	77
	19322	
	2	
	19324	

Exer.

3.

0	0	900 (9·65489384605
9	81	729
9	81	171000
9	162	155736
18	24300	15264000
9	1656	13896125
270	25956	1367875000
6	1692	1117933264
276	2764800	249941736
6	14425	223697856
282	2779225	26243880
6	14450	25168329
2880	279367500	1075551
5	115816	838952
2885	279483316	236599
5	115832	223720
2890	279599148	12879
5	2317	11186
28950	27962232	1693
4	2317	1678
28954	27964549	15
4	26	14
28958	2796481	1
4	26	
28962	2796507	

Exer.

4.	0	0	123'456'789 (497·93385922
	4	16	64
	4	16	59456
	4	32	53649
	8	4800	5807789
	4	1161	5114473
	120	5961	693316000
	9	1242	668132739
	129	720300	25183261
	9	10339	22312740
	138	730639	2870521
	9	10388	2231421
	1470	74102700	639100
	7	134271	595049
	1477	74236971	44051
	7	134352	37191
	1484	74371323	6860
	7	448	6694
	14910	7437580	166
	9	448	149
	14919	7438028	17
	9	4	15
	14928	743807	2
	9	4	
	14937	743811	

Exer.

5.

0			12'345'678 (231·1204185
2	4	8	
2	4	4345	
2	8	4167	
4	1200	178678	
2	189	159391	
60	1389	19287000	
3	198	16015231	
63	158700	3271769	
3	691	3204710	
66	159391	67059	
3	692	64100	
690	16008300	2959	
1	6931	1602	
691	16015231	1357	
1	6932	1282	
692	16022163	75	
1	139	80	
6930	1602355		
1	139		
6931	1602494		
1			
6932			
1			
6933			

Exer.

6.

0	0	1'234'567 (107·2765724
1	1	1
1	1	0234567
1	2	225043
2	30000	9524000
1	2149	6882248
300	32149	2641752
7	2198	2414860
307	3434700	226892
7	6424	207132
314	3441124	19760
7	6428	17262
3210	3447552	2498
2	225	2416
3212	344980	82
2	225	69
3214	345205	13
2	2	14
3216	34522	
	2	
	34524	

Exer.

7.

0	0	44·600 (3·546323
3	9	27
3	9	17600
3	18	15875
6	2700	1725000
3	475	1486864
90	3175	238136
5	500	225954
95	367500	12182
5	4216	11317
100	371716	865
5	4232	754
1050	375948	111
4	64	113
1054	37659	
4	64	
1058	37723	
4		
1062		

8. $4\times15^3=4\times225900$, the cube root of which (see *Exer.* 3) is 9·65489384605. Then, to find the answer, divide this by 15.

Exer. 9. $8 \times 9^2 = 8 \times 81 = 648$. Hence, the answer will be found by dividing the cube root of 648 by 9. The work for finding the cube root of 648 is as follows:—

0	0	648 (8·65349742187
8	64	512
8	64	136000
8	128	124056
16	19200	11944000
8	1476	11158625
240	20676	785375000
6	1512	673636077
246	2218800	111738923
6	12925	89853444
252	2231725	21885479
6	12950	20218167
2580	224467500	1667312
5	77859	1572540
2585	224545359	94772
5	77868	89859
2590	224623227	4913
5	1038	4493
25950	22463361	420
3	1038	225
25953	22464399	195
3	23	179
25956	2246463	16
3	23	15
25959	2246486	1

Exer.

10.

0	0	376 (7·2176522
7	49	343
7	49	33000
7	98	30248
14	14700	2752000
7	424	1557361
210	15124	1194639
2	428	1092721
212	1555200	101918
2	2161	93756
214	1557361	8162
2	2162	7813
2160	1559523	349
1	151	312
2161	156103	37
1	151	31
2162	156254	6
	1	
	15626	
	1	
	15627	

Exer.

11.

0	0	50 (3·6840315
3	9	27
3	9	23000
3	18	19656
6	2700	3344000
2	576	3180032
90	3276	163968
6	612	162684
96	388800	1284
6	8704	1221
102	397504	63
6	8768	41
1080	406272	22
8	44	20
1088	40671	2
8	44	
1096	40715	
8		
1104		

0	0	31 (3·1413807
3	9	27
3	9	4000
3	18	2791
6	2700	1209000
3	91	1168144
90	2791	40856
1	92	29588
91	288300	11268
1	3736	8879
92	292036	2389
1	3752	2368
930	295788	21
4	9	21
934	29588	
4	9	
938	29597	
4		
942		

0	0	(50+31 =) 81 (4·3267487
4	16	64
4	16	17000
4	32	15507
8	4800	1493000
4	369	1114568
120	5169	378432
3	378	336390
123	554700	42042
3	2584	39305
126	557284	2737
3	2588	2246
1290	559872	491
2	78	449
1292	56065	42
2	78	39
1294	56143	3
2	1	
1296	5615	
	1	
	5616	

Then, 3·6840315 + 3·1413807 = 6·8254122, and 6·8254122 − 4·3267487 = 2·4986635.

Exer.

12.

0	0	(50−31 =) 19 (	2·6684016
2	4		8
2	4		11000
2	8		9576
4	1200		1424000
2	396		1245096
60	1596		178904
6	432		170328
66	202800		8576
6	4716		8542
72	207516		34
6	4752		21
780	212268		13
6	64		13
786	21291		
6	64		
792	21355		
6			
798			

Now, from the last question we find the difference of the cube roots of 50 and 31 to be 0·5426508; and taking this from 2·6684016, the root just found, we get 2·1257508, the answer.

EXTRACTION OF ROOTS IN GENERAL.

(Arithmetic, page 266.)

Exer. 1.

0	0	0	0	987654321(19·27274478
1	1	1	1	1
1	1	1	1	977654321
3	4	5	6	883871739
4	5	6	7000000	93782582
6	10	15	91207971	67981030
10	15	210000	98207971	25801552
10	20	8034219	231113196	24817366
20	350000	10134219	329321167	984186
15	542691	15545025	1058398	717052
35000	892691	25679244	33990515	267134
25299	834534	26318835	1077026	251046
60299	1727225	51998079	35067541	16088
32427	1197090	9218	38584	14347
92726	2924315	529199	3545338	1741
40284	1636920	9314	38815	1435
133010	4561235	538513	3584153	306
48870	48	9410	111	287
181880	4609	547923	358526	19
53185	48	33	111	
240065	4657	5512	358637	
	48	33	4	
	4705	5545	35868	
	48	33		
	4753	5578		

In the foregoing work, the first and second columns have been omitted for want of room. The successive numbers in the first column are 0, 1, 1, 1, 2, 1, 3, 1, 4, 1, 5, 1, 6, 1, 70, 9, 79, 9, 88, 9, 97, 9, 106, 9, 115, 9, 124: and those in the second, 0, 1, 1, 2, 3, 3, 6, 4, 10, 5, 15, 6, 2100, 711, 2811, 792, 3603, 873, 4476, 954, 5430, 1035, 6465, 1116, 7581.

Exer.

2. This exercise might be solved by extracting the tenth root of 100, and the tenth root of the result. It will perhaps be rather simpler, however, to extract the fourth root of 100, the fifth root of that root, and the fifth root of the result. The work in the latter way is as follows:—

0	0	0	100 (3·162278
3	9	27	81
3	9	27	190000
3	18	81	113521
6	27	108000	7647900
3	27	5521	7360080
9	5400	113521	287820
3	121	5643	252672
120	5521	1191640	35148
1	122	35040	25294
121	5643	1226680	9854
1	123	35484	8853
122	5766	1262164	1001
1	74	120	1012
123	5840	126336	
1	74	120	
124	5914	126456	
	74	1	
	5988	12647	

0	0	0	0	3·162278 (1·258925
1	1	1	1	1
1	1	1	1	216227
1	2	3	4	148832
2	3	4	50000	6739580*
1	3	6	24416	5634375
3	6	10000	74416	1105205
1	4	2208	29264	989104
4	1000	12208	1036800*	116101
1	104	2424	90075	112851
50	1104	14632	1126875	3250
2	108	2648	93825	2511
52	1212	17280*	1220700	739
2	112	735	1568	628
54	1324	18015	123638	111
2	116	750	1576	
56	1440*	18765	125214	
2	3	765	18	
58	147	19530	12539	
2	3	1	18	
60*	150	196	12557	
	3	1		
	153	197		
	3	1		
	156	198		

* Here, for abbreviating the process, two figures are annexed in the fifth column, one in the fourth, and none in the third; while one figure is cut off in the second column, and two in the first. The like has been done also in other instances.

0	0	0	0	1·258925 (1·04712855
1	1	1	1	
1	1	1	1	25892500
1	2	3	4	21665280
2	3	4	5000000	4227220
1	3	6	416320	4149964
3	6	100000	5416320	77256
1	4	4080	432960	60094
4	1000	104080	5849280	17162
1	20	4160	7924	12021
50	1020	108240	592852	5141
	20	4240	7973	4808
	1040	112480	600825	333
	20	7	11	301
	1060	1132	60094	32
	20	7	11	30
	1080	1139	60105	2
		7		
		1146		

The method of working this question, by Rule II., will be as follows. It might be wrought also according to the method pointed out in the note in the next page.

We have here $n=100$: and consequently $n+1=101$, and $n-1=99$. By some trials, it will be found that the required root is about 1·047. The second power of this is 1·096209; the fourth, 1·201674171681; the eighth, 1·444020814885; the sixteenth, 2·085196113821; the thirty-second, 4·348042833095; the sixty-fourth, 18·905476478428; the ninety-sixth, 82·201821508274; and the hundredth, 98·779805771625. Then, adding the product of this by 101 to 9900, we obtain 19876·760382934125, and adding the product of the same power by 99 to 10100, we get 19879·200771390875; and, according to the rule, as 19876·760382934125 : 19879·200771390875 :: 1·047 : 1·047128546, the answer.

Exer.

3. This may be solved in the following manner, by finding the cube root of 1·05, and the fourth root of the result:—

0	0	1·05 (1·016397
1	1	1
1	1	50000
1	2	30301
2	30000	19699
1	301	18468
300	30301	1231
1	302	929
301	30603	302
1	18	279
302	3078	23
1	18	22
303	3096	1

0	0	0	1·016397 (1·00407427*
1	1	1	1
1	1	1	1639700
1	2	3	1609632
2	3	4000000	30068
1	3	2408	28338
3	60000	402408	1730
1	2	2416	1619
400	602	404824	111
	2		81
	604		30
	2		28
	606		2

* The extraction of roots of a high order by a single operation, by means of the first rule, in the manner in which it has thus far been employed, would be both laborious, and very inconvenient in practice. Thus, in the present exercise, there would be twelve columns, and in the preceding one a hundred: and it is only some of the latter of these, that would be necessary for finding the root, unless it were to be extracted to a great number of places. In such cases, if we employ but a single operation, we may find and use only as many of the numbers in the latter columns as may be necessary for the degree of accuracy required, omitting the earlier columns

Exer.

4. Since $9^{\frac{9}{5}}=9^{\frac{5}{5}}\times 9^{\frac{4}{5}}=9\times 9^{\frac{4}{5}}$, the answer will be found by multiplying the fifth root of the fourth power of 9 by 9. Now, the fourth power of 9 being 6561, the work for finding its fifth root is as follows:—

0	0	0	0	6561 (5·7995466
5	25	125	625	3125
5	25	125	625	343600000
5	50	375	2500	289192057
10	75	500	31250000	54407943
5	75	750	10063151	49025772
15	150	1250000	41313151	5382171
5	100	187593	11466854	5073030
20	25000	1437593	52780005	309141
5	1799	200529	169308	282755
250	26799	1638122	5447308	26386
7	1848	213808	171936	22624
257	28647	1851930	5619244	3762
7	1897	292	1745	3394
264	30544	18812	563670	368
7	1946	292	1745	339
271	32490	19104	565415	29
7		292	10	
278		19396	56551	
7			10	
285			56561	

Then, $5\cdot7995466\times 9=52\cdot1959194$, the answer.

---

altogether. In this way, the work of the present exercise will be as follows:

2200000	6600000	12000000	1·05 (1·004074
	9	2676	1
	669	122676	500000
	9	2712	490704
	678	125388	9296
			8777
			519
			501
			18

To find the headings in this mode proceed as in the margin,

EQUIDIFFERENT SERIES.

(Arithmetic, page 267.)

Exer. 1. $1\frac{1}{4} \times 53 = 66\frac{1}{4}$, and $100 - 66\frac{1}{4} = 33\frac{3}{4}$.

2. $3\frac{3}{8} \times 99 = 356\frac{2}{8}$, and $36 + 356\frac{2}{8} = 392\frac{2}{8}$.

3. $\frac{7}{8} \times 98 = 85\frac{3}{4}$, and $329 - 85\frac{3}{4} = 243\frac{1}{4}$.

4. $2\frac{1}{6} \times 366 = 793$, and $1000 - 793 = 207$, the less extreme. Then, $1000 + 207 = 1207$, and $1207 \times 367 = 442969$, the half of which is the required sum.

5. $\frac{1}{100} \times 50 = \frac{1}{2}$, and $1 - \frac{1}{2} = \frac{1}{2}$, the less extreme. Then, $1 + \frac{1}{2} = 1\frac{1}{2}$, and $1\frac{1}{2} \times 51 = 76\frac{1}{2}$, the half of which is the answer.

6. $500 - 70 = 430$; $430 \div 10 = 43$; and $43 + 1 = 44$.

7. $579 - 3 = 576$; $576 \div 9 = 64$; and $64 + 1 = 65$, the number of terms. Then, $579 + 3 = 582$; $582 \times 65 = 37830$; and $37830 \div 2 = 18915$, the sum of the series.

8. $1700 - 8 = 1692$; $1692 \div 12 = 141$; and $141 + 1 = 142$, the number of terms. Hence, rejecting 2 for the extremes, we have remaining 140, the number of the means.

---

making twelve additions in the first column, eleven in the second, ten in the third, &c. In this way, 12, 66, 220, &c., are obtained. Then, 12 is to be multiplied by the eleventh power of the first figure of the root (here 1); 66, by the tenth power of the same figure; 220, by its ninth power, &c. The results, 12, 66, 220, &c., taken in a reversed order, are the headings used above; and the twelfth power of 1, the first figure of the root, is what is to be taken from the given number, 105. Having thus obtained the headings, we annex four ciphers to the first, five to the second, and six to the third; while, to the last we annex five, making seven figures, with 05; the addition of these ciphers giving a sufficient degree of accuracy.

0	0	0
1	1	1
1	1	1
1	2	3
2	3	4
1	3	6
3	6	10
.....		
.....		
.....		
12	66	220

The mathematical reader will readily perceive, that if r be the first figure of the root, and if $1 + r$ be raised to the nth power, n denoting the order of the root, the terms of the result, except the first and last, will be the headings, and the nth power of r is what is to be taken from the given number. The raising of $1 + r$ to the nth power, when n is large, is most easily effected by means of the binomial theorem.

Exer.

9. $300-3=297$, and $10-1=9$. Then, $279\div 9=33$.

10. $1001-1=1000$, and $100001-1=100000$. Then, $100000\div 1000=100$.

11. Here, the number of terms is 7. Then, $7-1=6$, and $30-20=10$; also, $10\div 6=1\frac{2}{3}$, the common difference; and by adding this to 20, we get the first mean; by adding the same to that mean, we get the second, &c.

12. $49-4=45$; $6-1=5$; and $45\div 5=9$, the common difference. Then, $4+9=13$, the second term; $13+9=22$, the third, &c.

13. $576\times 2=1152$; $1152\div 24=48$; and $48-1=47$.

14. $1275\times 2=2550$; $2550\div 50=51$; and $51-3\frac{3}{4}=47\frac{1}{4}$.

15. $1+10000=10001$; $10001\times 10000=100010000$; $100010000\div 2=50005000$.

16. Here, the first term being 1, the common difference 2, and the number of terms 10000, the last term is found, by Rule I., to be 19999. Then, $1+19999=20000$; $20000\times 10000=200000000$; and $200000000\div 2=100000000$.

17. It is evident that the answer in this exercise will be exactly double of that in Exercise 15.

To solve it, however, without reference to that exercise, we have the first term $=2$, the common difference 2, and the number of terms 10000; and, by Rule I. we find the last term to be 20000. Then, $2+20000=20002$; $20002\times 10000=200020000$; and $200020000\div 2=100010000$.

18. The answer in this exercise is evidently treble of that in Exercise 15.

To work it, however, without reference to that exercise, the first term being 3, the common difference 3, and the number of terms 10000, we find the last term, by Rule I., to be 30000. Then, $3+30000=30003$; $30003\times 10000=300030000$; and $300030000\div 2=150015000$.

19. 30 days -1 day $=29$ days; $\frac{1}{4}\times 29=7\frac{1}{4}$ miles; and $30-7\frac{1}{4}=22\frac{3}{4}$ miles, the last term. Then, $30+22\frac{3}{4}=52\frac{3}{4}$; $52\frac{3}{4}\times 30=1582\frac{1}{2}$; and $1582\frac{1}{2}\div 2=791\frac{1}{4}$.

Exer.

20. To find the number of strokes in 12 hours, we have the first term 1, the last 12, and the number of terms 12. Then, $1 + 12 = 13$; $13 \times 12 = 156$; and $156 \div 2 = 78$, the number of strokes in 12 hours: by doubling which, and multiplying the result by 365, we obtain the answer.

21. Here the first term is evidently 12 yards, the common difference $2\frac{1}{2}$ yards, and the number of terms 120. Hence, by Rule I., the last term is found to be $309\frac{1}{2}$ yards. Then, $12 + 309\frac{1}{2} = 321\frac{1}{2}$; $321\frac{1}{2} \times 120 = 38580$; and $38580 \div 2 = 19290$ yards, the sum of the series; and this, by reduction, is found to be equivalent to 10 miles, 7 furlongs, 27 perches, $1\frac{1}{2}$ yard.

22. Here, the first term is 16 feet 1 inch, the common difference twice that space, and the number of terms 60; and the sum of the series would be found in the usual way.

The following method is rather easier. Consider the first term as 1, the common difference as 2, and the number of terms as 60. Then, $60 - 1 = 59$; $2 \times 59 = 118$; and $1 + 118 = 119$, the last term. Again, $1 + 119 = 120$; $120 \times 60 = 7200$; and $7200 \div 2 = 3600$, the sum of the series; and if 16 feet 1 inch be multiplied by this sum, the product will be the answer.

CONTINUAL PROPORTIONALS.

(Arithmetic, page 271.)

Exer. 1. By involution, the seventeenth power of 3 is 129140163; the product of which by 12 is the answer.

2. By involution, $1{\cdot}07^3 = 1{\cdot}1449$; $1{\cdot}07^4 = 1{\cdot}31079601$; $1{\cdot}07^8 = 1{\cdot}71818618$; $1{\cdot}07^{12} = 2{\cdot}25219159$; and $1{\cdot}07^{13} = 2{\cdot}40984500$. Then, $1 \div 2{\cdot}40984500 = {\cdot}4149644$.

3. The thirteenth power of 1·04 is 1·665074; see the third column of Table I. at the end of the Arithmetic. Then the rest of the work will stand thus:—

$$
\begin{array}{r|l}
1{\cdot}665074) \; 500{\cdot}000000 & \left\{ \begin{array}{l} 300{\cdot}28695, \text{ or} \\ 300{\cdot}287, \text{ nearly.} \end{array} \right. \\
\underline{4995222} & \\
477800 & \\
\underline{333015} & \\
144785 & \\
\underline{133206} & \\
11579 & \\
\underline{9990} & \\
1589 & \\
\underline{1499} & \\
90 & \\
83 &
\end{array}
$$

4. $2 \times 2 = 4$, the second power of 2; $4 \times 4 = 16$, the fourth power; $16 \times 16 = 256$, the eighth power; and $256 \times 256 = 65536$, the sixteenth power. Then, $65536 \div 2 = 32768$, the fifteenth term, and also the answer, as the first term is 1.

5. $3 \times 3 = 9$, the second power of 3; $9 \times 9 = 81$, the fourth power; $81 \times 81 = 6561$, the eighth power; and $6561 \times 6561 = 43046721$, the sixteenth power: by dividing which by 3, we get the fifteenth power, or answer.

6. $13{\cdot}16872428 \times \frac{2}{3} = 8{\cdot}77914952$, and $100 - 8{\cdot}77914952 = 91{\cdot}22085048$. Lastly, divide this by $\frac{1}{3}$ $(= 1 - \frac{2}{3})$, or multiply it by 3.

Exer.

7. $18{\cdot}42015 \times 1{\cdot}06 = 19{\cdot}525359$; $19{\cdot}525359 - 1 = 18{\cdot}525359$; $1{\cdot}06 - 1 = {\cdot}06$; and $18{\cdot}525359 \div {\cdot}06 = 308{\cdot}755983$.

8. $1 - \frac{1}{1{\cdot}04} = \frac{1{\cdot}04}{1{\cdot}04} - \frac{1}{1{\cdot}04} = \frac{{\cdot}04}{1{\cdot}04} = \frac{1}{26}$. Then, divide 100 by this, to find the answer.

9. $4^8 = 65536$; $65536 - 1 = 65535$; $4 - 1 = 3$. Then, $65535 \div 3 = 21845$, and $21845 \times 6 = 131070$, the required sum.

10. By Exercise 5, the fourth power of $\frac{1}{3}$ is $\frac{1}{81}$, and the eighth, $\frac{1}{6561}$. Then, $\frac{1}{6561} \times \frac{1}{81} = \frac{1}{531441}$. Take this from 1, and divide the remainder, $\frac{531440}{531441}$, by $\frac{2}{3}$ $(= 1 - \frac{1}{3})$, to find the answer.

11. $1\frac{1}{2} = \frac{3}{2}$. Now the sixteenth power of this is $\frac{43046721}{65536} = 656\frac{55105}{65536}$. (The numerator and denominator of the improper fraction are found in Exercises 5 and 4.) Then, taking 1 from this, and 1 from $1\frac{1}{2}$, and dividing the former remainder by the latter, we get $1311\frac{22337}{32768}$; or, by reducing the fraction to a decimal, 1311·68167114, &c.

12. Here, $\frac{5}{6} = {\cdot}8\dot{3}$, the second power of which is ${\cdot}69\dot{4}$. Then, ${\cdot}69\dot{4} \times {\cdot}69\dot{4} = {\cdot}48225308$, the fourth power; ${\cdot}48225308 \times {\cdot}48225308 = {\cdot}23256804$, the eighth power; and ${\cdot}23256804 \times {\cdot}48225308 = {\cdot}11215666$, the twelfth power. Again, $1 - {\cdot}11215666 = {\cdot}88784334$, and $1 - \frac{5}{6} = \frac{1}{6}$. Divide the former remainder by the latter (or multiply the former by 6), and multiply the result, 5·32706004, by 12: the product, 63·92472, is the answer.

13. Here, by the rule, the ratio will be the eighth root of 10. Now, the square root of 10 is 3·1622777; the square root of this, or the fourth root of 10, is 1·7782794; and the square root of this, or the required ratio, is 1·333521.

14. Here, the first mean is evidently the cube root of 2, and the second the square of that mean.

15. $\frac{1}{10} \times 100 = 10$, the square root of which is the required mean.

16. The eighteenth power of the ratio 3 is 387420489. (This is easily found from Exercise 5.) Subtracting 1 from this and from the ratio, and dividing the first re-

mainder by the second, we get 193710244; the product of which by 2, the first term, is 387420488, the sum of the series. Dividing this by 7680, we get 50445·376 pints; and dividing again by 8 and 8, we get successively 6305·672 gallons, and 788·209 bushels; the price of which, at 7*s.* per bushel, is £275·873, or £275 - 17 - 5½.

Exer.

17. Here the ratio is 1½, or ⅔; the twentieth power of which is $\frac{3486784401}{1048576}$, or $3325\frac{269201}{1048576}$. (This is easily found from Exercises 4 and 5.) Then, taking 1 from this, and dividing the remainder by ½ (= 1½−1), we get $6648\frac{269201}{524288}$, or 6648·51346; and multiplying this by the first term, 4, the product is the answer in shillings; which is to be divided by 20, to reduce it to pounds. Instead of these two operations, however, we might divide at once by 5. In either way, the quotient is £1329·70269, or £1329 - 14 - 0½, the answer.

18. By Exercise 4, the 32nd power of 2 is 4294967296, the product of which by itself is 18446744073709551616, the 64th power. If 1 be taken from this, the remainder is the sum of the series, since the ratio, diminished by 1, and the first term, are each 1. Dividing this by 10000 and by 112, we get 16470307208669·2425 cwt.; the price of which, at 12*s.* 6*d.* per cwt., is the answer.

19. The given decimal is equivalent to $\frac{4}{10}$, together with the infinite series $\frac{63}{1000}+\frac{63}{100000}+\frac{63}{10000000}+$&c., the ratio of which is $\frac{1}{100}$. To sum the infinite series, we divide $\frac{63}{1000}$ by $\frac{99}{100}$ $(=1-\frac{1}{100})$; and the quotient, $\frac{7}{110}$, is the required sum. Then, $\frac{4}{10}+\frac{7}{110}=\frac{51}{110}$, the answer.

20. Here, the given decimal is equivalent to $\frac{5}{10}$, or ½, together with the infinite series $\frac{185}{10000}+\frac{185}{10000000}+$&c., the ratio of which is $\frac{1}{1000}$. Then, for the sum of the infinite series, we have $1-\frac{1}{1000}=\frac{999}{1000}$, and $\frac{185}{10000}\div\frac{999}{1000}=\frac{185}{9990}$, which may be reduced to $\frac{1}{54}$. The sum of this and ½ is $\frac{14}{27}$, the answer.

21. Here, the ratio is ½; and dividing ¼ by $1-\frac{1}{2}$, we get ½, the answer.

22. Here, dividing ⅓ by $1-\frac{1}{3}$, we get ½, which is the required sum.

COMPOUND INTEREST.

(Arithmetic, page 276.)

Exer. 1. $1{\cdot}1^2 = 1{\cdot}21$; $1{\cdot}1^4 = 1{\cdot}4641$; $1{\cdot}1^8 = 2{\cdot}1435888$; and $2{\cdot}1435888 \times 1{\cdot}21 = 2{\cdot}593742$, the answer.

By multiplying the fourth power by 1·1, we should get 1·61051, the fifth power; and the answer would also be obtained by multiplying this by itself.

2. $1{\cdot}0625^2 = 1{\cdot}12890625$; $1{\cdot}0625^4 = 1{\cdot}27442932$; $1{\cdot}0625^8 = 1{\cdot}62417009$; $1{\cdot}0625^{16} = 2{\cdot}63792848$; $1{\cdot}0625^{17} = 2{\cdot}80279901$.

3, 4, 5, 6. In these exercises, the powers, up to the fiftieth, will be found in Table I.,* to which the teacher may have recourse. Then, in each exercise, the square of the fiftieth power is the hundredth.

7. $1{\cdot}05^9 = 1{\cdot}551328$. Then,

	1·551328
	£251 - 16 - 6
	1551328
	7756640
	3102656
10*s.* 0*d.* = £½.....	775664
4*s.* 0*d.* = £⅕.....	310266
2*s.* 6*d.* = ¼ of 10*s.*	193916
	£390·663174, or
	£390 - 13 - 3.

8. $1{\cdot}04^{15} = 1{\cdot}800944$; the product of which with £212 is £381·800128, or £381 - 16.

9. By Exercise 15 in involution, the sixteenth power of 1·055 is 2·35526272, and its second power is 1·113025; and dividing the former by the latter, we get 2·11609148, the fourteenth power. Then,

* In these exercises, and in those that follow, the tables at the end of the Arithmetic are referred to, as often as they can be employed; and they may be used in like manner by the teacher, when he may cause the learner to perform the work at full length.

2·11609148
214

846436592
211609148
423218296

£452·84357672
6s. 8d. = £⅓... ·70536383 } Subtract.

£452·13821289, or
£452 - 2 - 9¼.

Exer.

10. $1{\cdot}06^{12} = 2{\cdot}012196$. Then,

2·012196
£463 - 10 - 10

6036588
12073176
8048784
10s. 0d. = £½..... 1006098
10d. = 1/12 of 10s. 83841

£932·736687, or
£932 - 14 - 8¾.

11. $1{\cdot}045^2 = 1{\cdot}092025$; $1{\cdot}045^4 = 1{\cdot}19251860$; $1{\cdot}045^8 = 1{\cdot}42210062$; $1{\cdot}045^{16} = 2{\cdot}02237017$; and $1{\cdot}045^{17} = 2{\cdot}11337683$. Then,

2·11337683
£295 - 12 - 6

1056688415
1902039147
422675366
10s. 0d. = £½ 105668841
2s. 6d. = ¼ of 10s. 26417210

£624·76702536, or
£624 - 15 - 4.

12. By Exercise 14 in involution, the eighth power of 1·035 is 1·3168090; and thence we find the twelfth to be 1·5110685, and the thirteenth 1·5639559. Then,

1·5639559
£495 - 7 - 6

78197795
140756031
62558236
5s. 0d. = £¼..... 3909890
2s. 6d. = ½ of 5s. 1954945

£774·7446540, or
£774 - 14 - 10¾.

Exer.

13. $1{\cdot}05^{16} = 2{\cdot}182875$. Then,

2·182875
£649 - 13 - 6

19645875
8731500
13097250
10s. 0d. = £½..... 1091437
2s. 6d. = ¼ of 10s. 272859
1s. 0d. = $\frac{1}{10}$ of 10s. 109144

£1418·159315, or
£1418 - 3 - 2¼.

14. By Exercise 15 in involution, $1{\cdot}055^4 = 1{\cdot}23882465$; whence, $1{\cdot}055^5 = 1{\cdot}30696000$. Then,

1·30696
£582 - 7 - 6

261392
1045568
653480
5s. 0d. = £¼..... 32674
2s. 6d. = ½ of 5s. 16337

£761·14083, or
£761 - 2 - 9¾.

15. Here, the number of years is 9; and $1{\cdot}05^9 = 1{\cdot}551328$. Then,

1·551328
£1396 - 16 - 8
9307968
13961952
4653984
1551328
10*s.* 0*d.* = £½... 775664
6*s.* 8*d.* = £⅓... 517109
£2166·946661, or
£2166 - 18 - 11¼.

Exer.

16. Here, the time is 15 years; and, by Exercise 13 in involution, $1{\cdot}0475^{16} = 2{\cdot}10118587$; whence, by dividing by 1·0475, we find $1{\cdot}0475^{15} = 2{\cdot}00590587$. Then,

2·00590537
£648
1604724296
802362148
1203543222
£1299·82667976, or
£1299 - 16 - 6½.

17. $1\frac{1}{2} = 1{\cdot}2$; and $1{\cdot}2^2 = 1{\cdot}44$; $1{\cdot}2^4 = 2{\cdot}0736$; $1{\cdot}2^8 = 4{\cdot}29981696$; $1{\cdot}2^{16} = 18{\cdot}48842588$; and $1{\cdot}2^{32} = 341{\cdot}82189152$; by dividing which by 1·44, we get 237·37631356, the thirtieth power. Then, the product of this with £1200 is £284851·576272, or £284851 - 11 - 6¼, the answer.

18. £324 - 18 - 6 = £324·925; and $1{\cdot}05^9 = 1{\cdot}551328$. Then, the division by the contracted process, will be as follows:—

1·551328) 324·925000 (£209·4496, or
3102656 (£209 - 8 - 11⅜.
14659400
13961952
697448
620531
76917
62053
14864
13962
902

Exer.

19. £264 - 11 - 8 = £264·583. Again, by Exercise 13 in involution, $1{\cdot}0475^8 = 1{\cdot}44954678$; and $1{\cdot}0475^4 = 1{\cdot}20397126$; whence, $1{\cdot}0475^{12} = 1{\cdot}74521267$. Then, dividing £264·583 by this number, we get £151·6052, or £151 - 12 - 1¼, the answer.

20. £554 - 18 - 4=£554·916, and $1{\cdot}04^{27} = 2{\cdot}883369$. Then, if the former be divided by the latter, the quotient will be the answer.

21. $1{\cdot}08^2 = 1{\cdot}1664$; $1{\cdot}08^4 = 1{\cdot}36048896$; $1{\cdot}08^8 = 1{\cdot}85093021$; $1{\cdot}08^{16} = 3{\cdot}42594265$; $1{\cdot}08^{20} = 4{\cdot}66095715$; $1{\cdot}08^{27} = 5{\cdot}43654042$. Then, the answer will be found by dividing £1000 by this result.

22. By Exercise 11 of this rule, $1{\cdot}045^8 = 1{\cdot}42210062$; and dividing this by 1·045, we get $1{\cdot}36086183 = 1{\cdot}045^7$. Then, the answer is found by dividing £4500 by this amount.

23. 1⅛ = 1·125; the second power of which is 1·265625; its fourth power, 1·60180664; its eighth power, 2·56578451; and its twelfth power, 4·10989066. Then, if £15000 be divided by this, the quotient is the answer.

24. The present value of £1000 payable at present is £1000; of £1000 payable at the end of 5 years ($= £1000 \div 1{\cdot}04^5$) is £821 - 18 - 6½; and of £1000 payable at the end of 10 years ($= £1000 \div 1{\cdot}04^{10}$) is £675 - 11 - 3½. The sum of these is £2497 - 9 - 10. Again, the present value of £3000 (=£821 - 18 - 6½ × 3) is £2465 - 15 - 7½; which is to be taken from the foregoing sum to find the answer.

ANNUITIES CERTAIN.

(Arithmetic, page 282.)

Exer. 1. $1{\cdot}04^{10} = 1{\cdot}480244$; from which, if 1 be taken, there remains ·480244; and dividing this by ·04, we get 12·0061, the amount of an annuity of £1 for 10 years. Then, multiplying this by 100 brings out £1200·61, or £1200 - 12 - 2½, as the amount of an annuity of £100.

This might also be wrought by means of Table II. In this, the amount of £1 for 10 years, at 4 per cent. per annum, is found to be 12·006107; and if this be multiplied by 100, the answer is found to be £1200·6107, or £1200 - 12 - 2½, as before. In like manner, Exercises 2, 3, 4, and 8 may be wrought by means of the same table.

Exer.

2. Here, $1{\cdot}06^{14}-1 = 1{\cdot}260904$; and if this be divided by ·06, the quotient is 21·01506; the product of which into £13 - 15 - 9 will be the answer.

3. $1{\cdot}06^{9}-1 = {\cdot}689479$; ·689479÷·06 = 11·491316; and the product of this into £56 - 17 - 6 is the answer.

4. $1{\cdot}05^{12}-1 = {\cdot}795856$; ·795856 ÷ ·05 = 15·91712; which increase in the ratio of £1 to £11 - 7 - 6 to get the answer.

5. By Exercise 15 in involution, $1{\cdot}055^{8} - 1 = {\cdot}53468652$; ·53468652÷·055 = 9·721573; and 9·721573 × £34 - 2 - 6 = £331·748679, or £331 - 14 - 11¾, the answer.

6. By Exercise 16 in involution, $1{\cdot}07^{16}=2{\cdot}952163749$; and dividing this by 1·07, we get $1{\cdot}07^{15} = 2{\cdot}759031541$. Then, taking 1 from this, and dividing the remainder by ·07, we get 25·129022, the amount in pounds of an annuity of £1 for the 15 years. Then this amount is to be increased in the ratio of £1 to £14 - 15 - 9, to find the answer.

7. By Exercise 11 in compound interest, $1{\cdot}045^{16} = 2{\cdot}02237017$, and $1{\cdot}045^{2} = 1{\cdot}092025$; and dividing the former by the latter, we get $1{\cdot}851944937 = 1{\cdot}045^{14}$. Subtracting 1 from this, and dividing the remainder by ·045, we get 18·9321097; the product of which with £51 - 2 - 8½ is the answer.

8. Here, there are 34 payments, and $1{\cdot}03^{34}-1 = 1{\cdot}731905$, the quotient of which by ·03 is 57·73016; and this is to be multiplied by the half of 75, to find the answer in pounds.

9. $1{\cdot}1^{6}-1={\cdot}771561$; ·771561÷·1=7·71561; then twice £11 - 7 - 6 is to be multiplied by 7·71561.

Exer.

10. $5 \div 4 = 1\frac{1}{4} = 1{\cdot}25$; $101{\cdot}25 \div 100 = 1{\cdot}0125$. Then, $1{\cdot}0125^2 = 1{\cdot}02515625$; $1{\cdot}0125^4 = 1{\cdot}0509453370$; $1{\cdot}0125^8 = 1{\cdot}1044861014$; $1{\cdot}0125^{16} = 1{\cdot}2198895482$; $1{\cdot}0125^{32} = 1{\cdot}4881305097$; $1{\cdot}0125^{64} = 2{\cdot}2145324140$; and $1{\cdot}0125^{80} (= 1{\cdot}0125^{64} \times 1{\cdot}0125^{16}) = 2{\cdot}7014849461$. Taking 1 from this, and dividing the remainder by ·0125, we get 136·118795688. Multiply one fourth of £120 by 136·118795688 to find the answer.

11. $1{\cdot}05^9 = 1{\cdot}551328$; $1{\cdot}05^9 - 1 = {\cdot}551328$; ·551328 ÷ ·05 = 11·02656; and 11·02656 ÷ 1·551328 = 7·10728, the product of which with £84 - 7 - 9, is the answer.

12. $1{\cdot}06^{12} = 2{\cdot}012196$; $1{\cdot}06^{12} - 1 = 1{\cdot}012196$; 1·012196 ÷ ·06 = 16·86993; and 16·86993 ÷ 2·012196 = 8·383842. Then, to find the answer, multiply £46 - 12 - 9 by this.

13. $1{\cdot}025^2 = 1{\cdot}050625$; $1{\cdot}025^4 = 1{\cdot}10381289$; and $1{\cdot}025^6 = 1{\cdot}15969342$. Then, $1{\cdot}025^6 - 1 = {\cdot}15969342$; ·15969342 ÷ ·025 = 6·3877368; and 6·3877368 ÷ 1·15969342 = 5·508125; the product of which with £75 is the answer.

14. By Exercise 11 in involution, $1{\cdot}035^4 = 1{\cdot}1475230$; and thence $1{\cdot}035^5 = 1{\cdot}1876863$. Then, $1{\cdot}035^5 - 1 =$ ·1876863; ·1876863 ÷ ·035 = 5·362466; and 5·362466 ÷ 1·1876863 = 4·515052. The product of this with £58 - 10 is the answer.

15. $1{\cdot}04^{10} - 1 = {\cdot}480244$; ·480244 ÷ ·04 = 12·0061; and 12·0061 ÷ 1·480244 = 8·110893. Then, to find the answer, multiply £113 - 15 by this.

16. By Exercise 9 in this article, $1{\cdot}1^6 = 1{\cdot}771561$; and thence $1{\cdot}1^{12} = 3{\cdot}1384285$; and $1{\cdot}1^{13} = 3{\cdot}4522713$. Then, taking 1 from this, and dividing the remainder by ·1, we get 24·522713, dividing this by 3·4522713 we get 7·103356; and the product of 7·103356 with £95 - 5 - 6 is the answer.

17. $1{\cdot}05^{19} = 2{\cdot}526950$; $2{\cdot}526950 - 1 = 1{\cdot}526950$; 1·526950 ÷ ·05 = 30·539; and 30·539 ÷ 2·52695 = 12·08532; the product of which with £224 is the answer.

Exer.

18. The square root of 1·06 is 1·029563, the sixteenth power of which, or the eighth power of 1·06, is 1·593848. Then, taking 1 from each of these, and dividing the one result by the other, we get ·593848 ÷ ·029563 = 20·087542; and dividing this by 1·593848, we get 12·60317; the product of which with £20 is the answer.

19. £1 - 12 − 15*s.* = 17*s.* the reduction per acre; and the rent of 65 acres at this rate is £55 - 5, the annual reduction of the whole rent. Then, $1{\cdot}06^{36} - 1 = 7{\cdot}147252$; $7{\cdot}147252 \div {\cdot}06 = 119{\cdot}12086$; and $119{\cdot}12086 \div 8{\cdot}147252 = 14{\cdot}620987$; the product of which with £55 - 5 is the answer.

20. As $4\frac{3}{4}$: 100; or, as 19 : 400 :: £1 : £21 - 1 - $0\frac{3}{19}$ and as 19 : 400 :: £68 - 5 : £1436 - 16 - $10\frac{2}{19}$.

21. As $3\frac{1}{2}$: 100; or, as 7 : 200 :: £96 - 7 - 6 : £2753 - 11 - $5\frac{1}{7}$; and as 7 : 200 :: £1 : £28 - 11 - $5\frac{1}{7}$.

22. As 5 : 100 :: £1 : £20, the first answer.

Again, $\sqrt{1{\cdot}05} = 1{\cdot}0246950766$. Then, taking 1 from this, and dividing $\frac{1}{2}$ by the remainder, we get £20·24695; or £20 - 4 - $11\frac{1}{4}$, the second answer. (This would be found more easily by adding 1 to 1·024695, &c., and dividing the sum by ·05 × 2, or ·1.)

Lastly, $\sqrt{1{\cdot}0246950766} = 1{\cdot}012272234$. Then, if 1 be taken from this, and $\frac{1}{4}$ be divided by the remainder, we get £20·371189, or £20 - 7 - 5, the remaining answer.

23. *By Rule* III. 6 years + 8 years = 14 years. Then, by Exercise 9 in compound interest, $1{\cdot}055^{14} = 2{\cdot}11609148$; and taking 1 from this, and dividing the remainder by ·055, we get 20·292572; and if this be divided by 2·11609148, the quotient is 9·589648, the present value of an annuity of £1 for 14 years. Again, $1{\cdot}055^{6} = 1{\cdot}37884281$; $1{\cdot}37884281 - 1 = {\cdot}37884281$; ·37884281 ÷ ·055 = 6·888051; and 6·888051 ÷ 1·37884281 = 4·995530. Then, 9·589648 − 4·995530 = 4·594118, which is to be multiplied with £135 - 10 - 9, to find the answer.

By Rule IV. By Exercise 5 in this article, $1{\cdot}055^{8} = 1{\cdot}53468652$; and $1{\cdot}055^{8} - 1 \div {\cdot}055 = 9{\cdot}721573$. Then, $9{\cdot}721573 \div 1{\cdot}055^{8} = 6{\cdot}33456597$; and 6·33456597 ÷ 1·37884281 = 4·594118, as before.

By Rule V. $1{\cdot}055^{8} - 1 = {\cdot}53468652$; ·53468652 ÷ $1{\cdot}055^{14} = {\cdot}25267647$; and ·2526747 ÷ ·055 = 4·594118, the same present value as before.

Exer.

24. *By Rule* III. $1{\cdot}05^{10}-1={\cdot}628895$; ·628895÷·05 =12·5779; and 12·5779÷1·628895=7·721738. Again, $1{\cdot}05^{4}-1={\cdot}215506$; ·215506 ÷ ·05 = 4·31012; and 4·31012÷1·215506 = 3·545947. Taking this from the former result, we get 4·175791; the product of which with £79 - 12 - 6 will be the answer.

By Rule IV. $1{\cdot}05^{6}-1={\cdot}340096$; ·340096 ÷ ·05 = 6·80192; and 6·80192 ÷ 1·340096 = 5·075696. Then, $1{\cdot}05^{4}=1{\cdot}215506$; and 5·075696÷1·215506=4·175788, the same as before, nearly.

By Rule V. $1{\cdot}05^{6}-1={\cdot}340096$; $1{\cdot}05^{10}=1{\cdot}628895$; ·340096 ÷ 1·628895 = ·2087894; and ·2087894 ÷ ·05 = 4·175788, the same as by Rule IV.

25. *By Rule* III. From Exercise 11 in compound interest, it will be easily found, that $1{\cdot}045^{10}=1{\cdot}55296943$; $1{\cdot}045^{3}=1{\cdot}141166125$; and $1{\cdot}045^{7}=1{\cdot}360861837$. Then, ·55296943 ÷ ·045 = 12·2882095, and 12·2882095 ÷ 1·55296943 = 7·9127182; also, ·141166125 ÷ ·045 = 3·137025; 3·137025÷1·141166125=2·7489644. Then, 7·9127182 − 2·7489644 = 5·1637538; the product of which with £58 - 9 - 10 is the answer.

By Rule IV. $1{\cdot}045^{7}-1={\cdot}360861837$; ·360861837÷ ·045 = 8·0191519; and 8·0191519 ÷ 1·360861837 = 5·892701. Then, 5·892701÷1·141166125 = 5·1637538, as before.

By Rule V. ·360861837 ÷ 1·55296943 = ·23236892; and ·23236892÷·045 = 5·1637538, the present value of one pound, the same as before.

26. *By Rule* III. By Exercise 16 in involution, we have $1{\cdot}07^{4}=1{\cdot}31079601$; $1{\cdot}07^{8}=1{\cdot}71818618$; and thence $1{\cdot}07^{12}=2{\cdot}25219159$. Then, 1·25219159÷·07 = 17·8884513; and 17·8884513÷2·25219159=7·9426863. Again, ·31079601 ÷ ·07 = 4·439943; and 4·439943 ÷ 1·31079601 = 3·3872112. Then, 7·9426863−3·3872112 =4·5554751; by which £54 - 12 - 3 is to be multiplied, to find the answer.

By Rule IV. ·71818618 ÷ ·07 = 10·2598026; 10·2598026 ÷ 1·71818618 = 5·9712985; and 5·9712985 ÷1·31079601 = 4·555475, as before.

By Rule V. ·71818618 ÷ 2·25219159 = ·318883253; and ·318883253÷·07=4·555475, the same present value as before.

Exer.

27. *By Rule* III. $1 \div \cdot 07 = 14 \cdot 2857143$, the present value of a perpetuity of £1, possessed at present. Then, by the last exercise, $1 \cdot 07^8 - 1 = \cdot 71818618$; $\cdot 71818618 \div \cdot 07 = 10 \cdot 2598026$; and $10 \cdot 2598026 \div 1 \cdot 71818618 = 5 \cdot 9712985$. Lastly, $14 \cdot 2857143 - 5 \cdot 9712985 = 8 \cdot 3144158$, the product of which with £84 - 7 - 6 is the answer.

By Rule IV. The present value of the perpetuity is $14 \cdot 2857143$, as found already. Then, $14 \cdot 2857143 \div 1 \cdot 07^8 = 8 \cdot 3144158$, as before. Rule V. is not applicable in case of a perpetuity.

28. *By Rule* III. $1 \cdot 05^8 = 1 \cdot 477455$. Then, $1 \div \cdot 05 = 20$. Also, $\cdot 477455 \div \cdot 05 = 9 \cdot 5491$; and $9 \cdot 5491 \div 1 \cdot 477455 = 6 \cdot 463208$. Then, $20 - 6 \cdot 463208 = 13 \cdot 536792$; which is to be multiplied into £136 - 17 - 9, to find the answer.

By Rule IV. $20 \div 1 \cdot 477455 = 13 \cdot 536791$, as before.

29. $1 \cdot 0425^2 = 1 \cdot 08680625$; $1 \cdot 0425^4 = 1 \cdot 181147824$; $1 \cdot 0425^8 = 1 \cdot 395110182$; and $1 \cdot 0425^{16} = 1 \cdot 94633242$. Then, $1 \div \cdot 0425 = 23 \cdot 529412$; and $23 \cdot 529412 \div 1 \cdot 94633242 = 12 \cdot 089102$; which is to be multiplied into £71 - 13 - 3, to find the answer.

30. *By Rule* V. $1 \cdot 05^{25} = 3 \cdot 386355$; and $1 \cdot 05^{39} = 6 \cdot 704751$. Then, $3 \cdot 386355 - 1 = 2 \cdot 386355$; $2 \cdot 386355 \div \cdot 05 = 47 \cdot 7271$; and $47 \cdot 7271 \div 6 \cdot 704751 = 7 \cdot 1184001$; and to find the answer, multiply this into £112 - 10.

31. By Exercise 6 in compound interest, $1 \cdot 06^{100} = 339 \cdot 302083$. Then, $1 \div \cdot 06 = 16 \cdot \dot{6}$; and $16 \cdot \dot{6} \div 339 \cdot 302083 = \cdot 0491204$; the product of which with £60 is the answer.

LIFE ANNUITIES.

(Arithmetic, page 287.)

Exer. 1. By Table IV. the present value of an annuity of £1, at 5 per cent., on the life of a person aged 38 years, is 13·805: the product of which by 138·5 is 1911·9925; that is, £1911 - 19 - 10¼.

2. Here the present value by Table IV. is 16·774; and the product of this by 180 is 3019·32; that is, £3019 - 6 - 4¾.

3. £1 - 14 - 6 − 10*s.* = £1 - 4 - 6; and the value of 28 acres, at £1 - 4 - 6 per acre, is £34 - 6. Then, by Table IV., the present value of £1 is £13·287; and the answer will be found by multiplying this by 34·3.

4. By Table V. the value for 25 and 70 years is 6·033; and for 25 and 75 it is 4·671. The difference of these is 1·362; by taking three fifths of which we get ·817; and subtracting this from 6·033, we find 5·216; the product of which by 39·5 (for £39 - 10) is the answer.

5. By Table V. the values for 35 and 40 years and for 40 and 40 years are 12·854 and 12·348; by adding one fifth of the difference of which to 12·348 we get 12·4492, the value for 39 and 40 years.

Secondly, the values for 35 and 45 years, and for 40 and 45 years, are 12·064 and 11·656; one fifth of the difference of which being added to 11·656, the sum is 11·7376 for 39 and 45 years.

Thirdly, by adding one fifth of the difference of these sums to the latter we obtain 11·87992, the value for 39 and 44 years.

Lastly, the values in Table IV. for 39 and 44 years are 15·358 and 14·162, the sum of which is 29·520. Taking 11·87992 from this we get 17·64008; the product of which with £200 is the answer.

6. The value for 50 and 55 years is 8·908. Then by inspecting Table IV., under 4 per cent., it will be seen that 8·908 is the value of a single life of about 62 years.

Secondly, by Table V. the values for 20 and 60 years, and for 20 and 65 years, are 8·942, and 7·508; by taking two fifths of the difference of which from the former we get 8·3684, the joint value for 20 and 62 years, or for 20, 50, and 55 years.

Thirdly, by Table V. the values for 20 and 50 years, and for 20 and 55 years, are 11·615 and 10·338.

Fourthly, the sum of the three joint values above found, 8·908, 11·615, and 10·338, is 30·861.

Fifthly, the values of the three single lives given in the question are, by Table IV., 11·043, 12·536, and 18·644; the sum of which, and of their joint value 8·368, already found, is 50·591.

Lastly, from this sum take 30·861, and there will remain 19·730, which is to be multiplied by 320 to find the answer in pounds.

Exer.

7. By Table IV. the values of two lives of 30 and 70 years are 17·131 and 6·293; the sum of which is 23·424. Also, by Table V. their joint value is 5·992; the difference between which and the sum last found is 17·432, the value on the longer of the two lives. Again, $1 \div \cdot 04 = 25$, the value of a perpetuity of £1. Taking 17·432 from this, we get 7·568, the value of £1 in reversion, which is to be multiplied by 400 to find the answer.

8. Here the value of £1 in perpetuity is $1 \div \cdot 03$, or $33\cdot\dot{3}$; and by Table IV. the value of £1 on a life of 66 years is 8·064. The difference of these is $25\cdot269\dot{3}$; and $25\cdot269\dot{3} \div 34\cdot\dot{3} = \cdot736$; also $\cdot736 \div 8\cdot064 = \cdot091269$

9. By Table IV. the value of a life of 45 years is 15·594; the double of which is 31·188. From this take 12·162, the joint value of two lives of 45, and there remains 19·026, the value of £1 on the longer of the two lives. Here also the value of the perpetuity of £1 is $33\cdot\dot{3}$; and taking 19·026 from this, we get $14\cdot3073\dot{3}$. Then $14\cdot307 \div 34\cdot\dot{3} = \cdot41672$, and $\cdot41672 \div 19\cdot026 = \cdot0219026$; the product of which by £4000 is the answer.

CONTINUED FRACTIONS.

(Arithmetic, page 296.)

Exer. 1. In this exercise, the several quotients are 2, 1, 4, 3, 2, 2, 1, and 30. Then,

2	1	4	3	2	2	1	30
	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{5}{14}$	$\frac{16}{45}$	$\frac{37}{104}$	$\frac{90}{253}$	$\frac{127}{357}$.

2. Here, the quotients are 2, 3, 3, 3, 2, 7, 1, 1, 1, and 2. Then,

2	3	3	3	2	7	1	1	1	2
	$\frac{1}{2}$	$\frac{3}{7}$	$\frac{10}{23}$	$\frac{33}{76}$	$\frac{76}{175}$	$\frac{565}{1301}$	$\frac{641}{1476}$	$\frac{1206}{2777}$	$\frac{1847}{4253}$.

3. The quotients are 3, 1, 1, 1, 1, 6, 1, 1, 1, 1, 6, &c. Then,

3	1	1	1	1	6	1	1	1	1	6	&c.
	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{2}{7}$	$\frac{3}{11}$	$\frac{5}{18}$	$\frac{33}{119}$	$\frac{38}{137}$	$\frac{71}{256}$	$\frac{109}{393}$	$\frac{180}{649}$	$\frac{1189}{4287}$, &c.

4. Here the quotients are 13, 1, 1, 2, 9, 1, 1, 2. Then,

13	1	1	2	9	1	1	2	
	$\frac{1}{13}$	$\frac{1}{14}$	$\frac{2}{27}$	$\frac{5}{68}$	$\frac{47}{639}$	$\frac{52}{707}$	$\frac{99}{1346}$	$\frac{250}{3399}$.

SCALES OF NOTATION.

(Arithmetic, page 233.)

Exer.

1. 3) 1000000
   3) 333333 1
   3) 111111 0
   3) 37037 0
   3) 12345 2
   3) 4115 0
   3) 1371 2
   3) 457 0
   3) 152 1
   3) 50 2
   3) 16 2
   3) 5 1
   1 2

Hence, the answer in the ternary scale is 1212210202001.

Again:

9) 1000000
9) 111111 1
9) 12345 6
9) 1371 6
9) 152 3
9) 16 8
1 7

Hence, in the nonary scale, the answer is 1783661.

Exer.

2. 1000000
   4
   4
   4
   16
   4
   64
   4
   256
   4
   1024
   4

4096, the answer in the decimal scale. This operation is evidently nothing else than finding the sixth power of 4.

Again, by dividing 4096 by 2, the quotient by 2, &c. we get 1 for the last quotient, after twelve divisions without remainder. Hence, the expression in the binary scale is 1000000000000.

Exer.

3.

123454321
6
8
6
51
6
310
6
1865
6
11194
6
67167
6
403004
6
12) 2418025
12) 201502 ... 1
12) 16791 ...10
12) 1399 ... 3
12) 116 ... 7
9 ... 8

Hence, the expression in the duodenary scale is 9873D1.

This might also be wrought thus:—

12) 123454321
12) 4152514 ... 1
12) 205429 ...10
12) 10251 ... 3
12) 312 ... 7
12) 13 ... 8
0 ... 9

In this method, since the given number is in the senary scale, each remainder is multiplied by 6, and the product is increased by the following figure.

Exer.

4.

12) 476897
12) 39741 ... 5
12) 3311 ... 9
12) 275 ... 11
12) 23 ... 11
1 ... 10

Hence, the answer is 1 DHH 95.

5.

2) 6666
2) 3333 ... 0
2) 1666 ... 1
2) 833 ... 0
2) 416 ... 1
2) 208 ... 0
2) 104 ... 0
2) 52 ... 0
2) 26 ... 0
2) 13 ... 0
2) 6 ... 1
2) 3 ... 0
1 ... 1

Hence, the answer in the binary scale is 1101000001010.

Again:—

5) 6666
5) 1333 ... 1
5) 266 ... 3
5) 53 ... 1
5) 10 ... 3
2 ... 0

Hence, the answer in the quinary scale is 203131.

Exer. 6.

$$
\begin{array}{rll}
 & 13579 & \\
 & 12 & \\
 & \overline{15} & \\
 & 12 & \\
 & \overline{185} & \\
 & 12 & \\
 & \overline{2227} & \\
 & 12 & \\
11\,) & \overline{26733} & \\
11\,) & \overline{2430} & \ldots\ 3 \\
11\,) & \overline{220} & \ldots\ 10 \\
11\,) & \overline{20} & \ldots\ 0 \\
 & \overline{1} & \ldots\ 9
\end{array}
$$

Hence, the answer is 190D3. This might also be wrought by dividing by 11, and multiplying each remainder by 12.

MISCELLANEOUS QUESTIONS.

(Arithmetic, page 304.)

Exer. 1. $6\frac{1}{2}d. \div 8 = \frac{13}{16}d.$, the price of each apple at the first rate; and $2\frac{1}{2}d. \div 3 = \frac{5}{6}d.$, the price of each at the second rate. Then, as $\frac{13}{16}d. : \frac{5}{6}d. : 108\frac{1}{3} : 111\frac{1}{9}$; and $111\frac{1}{9} - 100 = 11\frac{1}{9}$, the answer.

2. The first cost of each is evidently $\frac{1}{5}d.$ Then, as $100 : 140 :: \frac{1}{5}d. : \frac{7}{25}d.$, the selling price of each; which is obviously at the rate of 25 for $7d.$

3. The cost price is $112\frac{1}{2}d.$ Ten per cent. added to this makes $123\frac{3}{4}d.$ Suppose the whole 150 apples to be sold at 3 for $2\frac{1}{2}d.$: this would bring $125d.$, or $1\frac{1}{4}d.$ too much. Now these two rates are $\frac{20}{45}$ of a penny and $\frac{40}{45}$ of a penny per apple. The difference is $\frac{1}{45}$; and 60 times this $\frac{5}{4}d.$ or $1\frac{1}{4}d.$ Hence 60 apples must go at the smaller rate and 90 at the larger.

4. By doubling the mean, £52 - 10, we get £105, the sum of the extremes; and by taking from this the given extreme, £20, we get £85, the other extreme. Hence,

by Rule III. Arithmetic, page 268, £85 − £20 = £65; £65 ÷ £5 = £13; and 13 + 1 = 14.

Exer.

5. $1 \div 2 = \frac{1}{2}$; $1 + 2\frac{7}{8} = \frac{8}{23}$; and $1 \div 3\frac{1}{2} = \frac{2}{7}$, the reciprocals of the given distances. Then, as $\frac{1}{2} + \frac{8}{23} + \frac{2}{7} : \frac{1}{2}$; or, by reduction to a common denominator, and by using only the numerators, as 161 + 112 + 92, or 365 : 161 :: £164 - 5 : £72 - 9; as 365 : 112 :: £164 - 5 : £50 - 8; and as 365 : 92 :: £164 - 5 : £41 - 8.

6. 90 pair of gloves at 2*s.* 6*d.* per pair would bring 225*s.*, or £11 - 5, which is 25*s.* less than £12 - 10. The stockings cost 6*d.* per pair more than the gloves. Twice 25 sixpences are wanted to make up the whole amount received (£12 - 10). Then there must be 50 pair of stockings and 90 − 50 = 40 pair of gloves.

7. $12 \times \frac{5}{8} = 7\frac{1}{2}$, and $12 + 7\frac{1}{2} = 19\frac{1}{2}$. Now, by the question, the sum of this and five eighths of the father's age, is equal to the father's age; consequently, $19\frac{1}{2}$ must be three eighths of the father's age. Hence, as $\frac{3}{8} : 1$, or as 3 : 8 :: $19\frac{1}{2}$: 52, the answer.

8. By dividing the gains by the respective times, we get £22 - 16, £16 - 13, and £13 - 15; the sum of which is £53 - 4. Then,

As £53 - 4 : £1064; or, by contraction,
1 : 20 :: £22 - 16 : £456;
1 : 20 :: £16 - 13 : £333;
1 : 20 :: £13 - 15 : £275.

9. By dividing the gains by the times, we get £14 - 8 - 9, £20 - 9 - 6, and £26 - 5; the difference of the second and third of which is £5 - 15 - 6. Then,

As £5 - 15 - 6 : £220; or, by contraction,
21 : 800 :: £14 - 8 - 9 : £550;
21 : 800 :: £20 - 9 - 6 : £780;
21 : 800 :: £26 - 5 - 0 : £1000.

10. By assuming 1 as a common mean, and $1\frac{1}{2}$ as the ratio, we obtain for the series $\frac{4}{9}$, $\frac{2}{3}$, 1, $1\frac{1}{2}$, and $2\frac{1}{4}$; and by assuming the same mean, and $2\frac{1}{2}$ as the ratio, we find for the series $\frac{4}{25}$, $\frac{2}{5}$, 1, $2\frac{1}{2}$, and $6\frac{1}{4}$. The sum of all

these terms is $16\frac{77}{450}$. Then, as $16\frac{77}{450} : 80\frac{77}{90} :: 1 : 5$, the common mean. The other terms will be found by multiplying and dividing this mean by the ratios; or more easily by multiplying the several terms in the two series above found by 5.

Exer.

11. Here, according to the method shown in the Arithmetic, Question 13, page 302, we add together $\frac{1}{18}$, $\frac{1}{20}$, $\frac{1}{24}$, and $\frac{1}{27}$, and we divide the sum, $\frac{199}{1080}$, by 3; the quotient is $\frac{199}{3240}$. Then, as $\frac{199}{3240} : 1$, or as $199 : 3240 ::$ 1 day : $16\frac{56}{199}$ days, the time in which the work would be done by all.

Again, $\frac{199}{3240} - \frac{1}{20} = \frac{37}{3240}$; $\frac{199}{3240} - \frac{1}{24} = \frac{64}{3240}$; $\frac{199}{3240} - \frac{1}{27} = \frac{79}{3240}$; and $\frac{199}{3240} - \frac{1}{18} = \frac{19}{3240}$. Then, as $37 : 3240 ::$ 1 day : $87\frac{3}{37}$ days; as $64 : 3240 ::$ 1 day : $50\frac{5}{8}$ days; as $79 : 3240 ::$ 1 day : $41\frac{1}{79}$ days; and as $19 : 3240 ::$ 1 day : $170\frac{10}{19}$ days.

12. $13+16=29$; $13\times16=208$; and $208\div29=7\frac{5}{29}$ days, the answer.—(*See Arithmetic, page* 302, *Exercise* 12.)

13. By adding together $\frac{1}{21}$, $\frac{1}{24}$, $\frac{1}{28}$, and $\frac{1}{32}$, we get $\frac{5}{32}$, the part of the work done by all working two days and a half; and as $2\frac{1}{2}$ days : 1 day $:: \frac{5}{32} : \frac{1}{16}$, the part done by them in one day; and, consequently, they would finish the whole in 16 days.

Now, since $\frac{1}{21}$ of the work contains the part done by A and B in one day, and by C in half a day; and since $\frac{1}{28}$ contains the parts done by C and D in one day, and by A in half a day; it is evident that $\frac{11}{84}$, the sum of the latter, and twice the former, will contain the parts done by A in $2\frac{1}{2}$ days, by B in 2 days, by C in 2 days, and by D in 1 day. Again, $\frac{1}{24}$ of the work contains the parts done by B and C in one day, and by D in half a day; and, consequently, $\frac{1}{12}$, the double of this, contains the work done by B and C in 2 days, and by D in 1 day. Hence, by taking $\frac{1}{12}$ from $\frac{11}{84}$, we have remaining $\frac{1}{21}$, the part of the work done by A in $2\frac{1}{2}$ days; and, therefore, as $\frac{1}{21} : 1$, or as $1 : 21 :: 2\frac{1}{2}$ days : $52\frac{1}{2}$ days, the time required by A.

In like manner, by taking twice $\frac{1}{28}$ from the sum of $\frac{1}{32}$ and twice $\frac{1}{24}$, we get $\frac{29}{672}$, the part done by B in $2\frac{1}{2}$

days; by taking twice $\frac{1}{3\frac{1}{2}}$ from the sum of $\frac{1}{2\frac{1}{1}}$ and twice $\frac{1}{2\frac{1}{8}}$, we obtain $\frac{19}{336}$, the part done by C in $2\frac{1}{2}$ days; and, by taking twice $\frac{1}{2\frac{1}{1}}$ from the sum of $\frac{1}{2\frac{1}{4}}$ and twice $\frac{1}{3\frac{1}{2}}$, we get $\frac{1}{11\frac{1}{2}}$, the part done by D in $2\frac{1}{2}$ days. Then, as $29 : 672 :: 2\frac{1}{2}$ days : $57\frac{27}{29}$ days; as $19 : 336 :: 2\frac{1}{2}$ days : $44\frac{4}{19}$ days; and as $1 : 112 :: 2\frac{1}{2}$ days : 280 days.

Exer.

14. By dividing the gains by their respective stocks, we get $\frac{8}{25}$, $\frac{2}{5}$, and $\frac{12}{25}$. Then, $\frac{2}{5} - \frac{8}{25} = \frac{2}{25}$; and as $\frac{2}{25} : \frac{8}{25} :: 2$ months : 8 months; as $\frac{2}{25} : \frac{2}{5} :: 2$ months : 10 months; and as $\frac{2}{25} : \frac{12}{25} :: 2$ months : 12 months.

15. The given number is evidently equal to 5, together with 9 times the radix of the scale. Hence, taking 5 from the given number, and dividing the remainder by 9, we get the answer.

16. Water is composed of two volumes of H and one of O. The weight of one volume of O is 15·96 times the weight of one volume of H. Therefore the weight of 2 volumes of H, and one of O, is 17·96.

$$\left.\begin{array}{l}\text{Then,}\quad 709 \times \frac{15\cdot96}{17\cdot96} = 630\cdot047\\ \text{And,}\quad 11315 \times \frac{2\cdot0}{17\cdot96} = 1260\cdot022\end{array}\right\} 1890\tfrac{1}{16} \text{ approx.}$$

17. In following the method pointed out in the Arithmetic, page 226, we find the quotients to be 2, 3, 3, 3, 1, 1, 3, 6, 3, 1, &c. Then,

2	3	3	3	1	1	3	6	3	1	&c.
$\frac{1}{2}$	$\frac{3}{7}$	$\frac{10}{23}$	$\frac{33}{76}$	$\frac{43}{99}$	$\frac{76}{175}$	$\frac{271}{624}$	$\frac{1702}{3919}$	$\frac{5377}{12381}$	$\frac{7079}{16300}$,	&c.

18. The square root of 5 is 2·236068, and that of 11 is 3·3166248; and the quotients, found in the usual way, are 1, 2, 14, 2, 2, 2, 14, &c. Then,

1	2	14	2	2	2	14	&c.
$\frac{1}{1}$	$\frac{2}{3}$	$\frac{29}{43}$	$\frac{60}{89}$	$\frac{149}{221}$	$\frac{358}{531}$	$\frac{5161}{7655}$,	&c.

19. Here, it is evident, that the interest of the bill, during the required time, at 6 per cent., is equal to the interest of the false present worth for the same time at $6\frac{1}{2}$ per cent. Hence, it is obvious, that the bill must exceed the false present worth in the ratio of $6\frac{1}{2}$ to 6, or of 13 to 12; and, consequently, that the false discount, or the interest of the bill, must have been $\frac{1}{13}$ of the bill. Now, the interest of any sum for a year, being $\frac{6}{100}$ of the same sum, we have $\frac{6}{100} : \frac{1}{13}$; or, by reduction, $78 : 100 :: 1$ year : 1 year, $102\frac{37}{39}$ days.

Exer.

20. $100 - 3\frac{3}{4} = 96\frac{1}{4}$. Then, as $3\frac{3}{4} : 96\frac{1}{4} :: £54 : £1386$; and $£1386 - £5 = £1381$.

21. This question may be wrought, as in the margin, in the same manner as a common fraction is reduced to a decimal fraction, in the decimal scale. In the septenary scale, the denominator 9 is expressed by 12. We then find that this is contained 3 times in 50 (the numerator with a cipher annexed); and multiplying by 3, we get 36. In subtracting this, as we cannot take 6 from 0, we take it from 7, the radix of the scale, and carry 1. A cipher being annexed to the remainder, 12 is contained 6 times in the result. Then, in multiplying, we have 6 times 2 equal to 12, which contains one 7 with the remainder 5; we set down 5 therefore, and carry 1. Then, multiplying 1 by 6 and adding 1, we get 7, which in the septenary scale is expressed by 10. Proceeding thus, we get the remainder 5 after the third figure placed in the quotient; and this being the same as the given numerator, it is plain that the fraction is periodical, the period consisting of the three figures 361.

```
12 ) 5·0 ( ·361, &c.
     36
     ---
     110
     105
     ---
      20
      12
      --
       5, &c.
```

This method will be illustrated by the annexed process, in which the decimal notation is employed; and, instead of annexing a cipher, we multiply by 7, which, in the septenary scale, is equivalent.

```
    5
    7  } multiply.
9 ) 35 ( 3
    8
    7  } multiply.
9 ) 56 ( 6
    2
    7  } multiply.
9 ) 14 ( 1
    5, &c.
```

We might also obtain the answer by finding a power

of 7, from which if 1 be taken, the remainder will be divisible by the given denominator 9. This will be readily found to be the third power, 343. Then, $\frac{5}{9}$ of 342=190; which, in the septenary scale, is expressed by 361, the *period* of the required expression.

Exer.

22. $1\frac{1}{2}$=1·2. Then, $1{\cdot}2^{15}$=15·40702157. (This is easily found from Exercise 17, compound interest.) Taking 1 from this, and dividing the remainder by ·2, we get 72·0351078; the product of which into £400 is £28814·04312, the amount of an annuity of £400 at the end of 15 years, at the proposed rate of increase. Adding £12000 to this, we get £40814·04312, the sum which the merchant would have been worth, had there been no expenditure. Then, dividing this by $1{\cdot}2^{15}$, we get £2649·054, or £2649 - 1 - 1, the answer.

23. It is evident that, in every case, the drawing off of one gallon from the cask, when full, leaves in it $\frac{9}{10}$ of its previous contents. Hence, the quantity of rum left the first day is $\frac{9}{10}$ of 10 gallons; the second day, $\frac{9}{10}$ of that; and so on, till, at the twentieth day, it is only 10 gallons multiplied by the twentieth power of $\frac{9}{10}$; and, if this quantity be taken from 10 gallons, the remainder will be the quantity of water. By similar reasoning it would be shown, that the quantity of water contained in the cask, at the end of the second period of twenty days, would be equal to the quantity last mentioned, multiplied also by the twentieth power of $\frac{9}{10}$. Now, the twentieth power of $\frac{9}{10}$, or ·9, is ·12157665459. ($\cdot 9^2 = \cdot 81$; $= \cdot 6561$; $\cdot 9^8 = \cdot 43046721$; and $\cdot 9^{16} = \cdot 185302018885$.) The product of this by 10 being taken from 10, the remainder is 8·7842334541, the quantity of water in the cask at the end of twenty days; and the product of this by $\cdot 9^{20}$ is 1·0679577, &c. gallon, the answer.

24. It will appear, on consideration, that the required prices are two mean proportionals between the given ones. Hence, according to Rule IV. Arithmetic, page 273, we divide £391 - 11 - 10 by £246 - 12, or 93982*d.* by 59184*d.*; the quotient is $\frac{2\,3\,9\,8\,2}{5\,9\,1\,8\,4}$, or, in its lowest terms, $\frac{3\,4\,3}{2\,1\,6}$; and the cube root of this fraction is $\frac{7}{6}$, or $1\frac{1}{6}$, the ratio. Then, adding to £246 - 12 one sixth

of itself, we find one answer; and if to that answer one sixth of itself be added, the sum is the second answer.

Exer.

25. It is evident, that in $22\frac{1}{2}$ days, A travels as much as B would travel in $22\frac{1}{2}$ days, and in twice 9 days; that is, in $40\frac{1}{2}$ days. Hence, as $40\frac{1}{2}$ days : $22\frac{1}{2}$ days :: 18 miles : 10 miles, the answer.

26. $100+50=150$, and $150\div100=1\frac{1}{2}$. Then, the fourth power of $1\frac{1}{2}$ is $5\frac{1}{16}$; and taking 1 from this, and dividing the remainder by $1\frac{1}{2}-1$, we get $8\frac{1}{3}$; the product of which, into £300, is £2437½, the amount of an annuity of £300 forborn 4 years, at 50 per cent. per annum. Now, had there been no expenditure, each pound of the original capital would have amounted to $£5\frac{1}{16}$, while in reality it amounts only to £4. The difference of these is $£1\frac{1}{16}$, the loss on each pound of the original capital, in consequence of the yearly expenditure. But the value of the annuity we have found to be £2437½; and therefore, as $1\frac{1}{16}$: 1, or as 17 : 16 :: £2437½ : $£2294\frac{2}{17}$, the answer.

27. It is evident, from the nature of the question, that if the time from the end of the evening twilight till day-break, be divided into seven equal parts, the evening and morning twilight will each consist of two such parts, and the day from sunrise to sunset of eight such parts; the time also from noon till sunset, will consist of eight of the same parts. Hence, as $7+2+2+8$: 4 :: 24 hours : 5 hours, $3\frac{3}{19}$ minutes, the time, after noon, of setting.

28. By repeated multiplications by 30, it is found that the expression for the given number in the decimal scale is 895791; and from this the answer is obtained by continual divisions by 12.

29. The cube root of 2 is 1·259921. Then using this and unity, we find the quotients by the usual division to be 1, 3, 1, 5, 1, 1, 4, 1, 1, &c. Then, the rest of the work will stand as follows:—

1	3	1	5	1	1	4	1	1	&c.
	$\frac{1}{1}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{23}{29}$	$\frac{27}{34}$	$\frac{50}{63}$	$\frac{227}{286}$	$\frac{277}{349}$	$\frac{504}{635}$, &c.

Exer.

30. By reducing 644 and 1000 in the octary scale to the decimal scale, we get 420 and 512. Hence, it is evident that the fraction in the form in which it is given, is a decreasing infinite series of continual proportionals, whose first term is $\frac{420}{511}$, and ratio $\frac{1}{512}$; and the sum of this series is found, by Rule II., Arithmetic, page 271, to be $\frac{420}{511}$, or $\frac{60}{73}$.

31. By reducing $\frac{4}{13}$ in the quinary scale, and $\frac{7}{21}$ in the nonary scale, to the decimal scale, we get $\frac{4}{7}$ and $\frac{7}{19}$; the difference of which is $\frac{27}{133}$, the answer.

32. By reducing $\frac{54}{7}$ in the duodecimal scale, and $\frac{7}{54}$ in the octary scale, to the decimal scale, we get $\frac{64}{7}$ and $\frac{7}{44}$: the product of which is $1\frac{5}{11}$, the answer.

33. Since the time is four years, we have 124 : 24 :: £1 : $£\frac{6}{31}$, the true discount of £1 for that time, at 6 per cent. per annum; and as 116 : 16 :: £1 : $£\frac{4}{29}$, the true discount of £1 for the same time, at 4 per cent. per annum. Then, as $£\frac{6}{31} - £\frac{4}{29}$: £5 :: £1 : £89 - 18.

34. Here, if the whole quantity be supposed to consist of 12 equal parts, the three portions mentioned in the question will respectively consist of 4, 3, and 5 such parts. Then, $4\times1+3\times2+5\times3=25$; and as 25 : 12 :: 20 per cent. : $9\frac{3}{5}$ per cent. the first rate; which is to be doubled and trebled to find the other rates.

35. On each of the first, second, third, and fourth days, the person must have travelled four miles more than on each of the fifth, sixth, seventh, and eighth, respectively; or, on the whole, 16 miles more in the first four days, than in the second four. Hence, since he travels as much during the first four, as during the last five days, it is evident that he must have travelled 16 miles on the ninth day. Taking this, therefore, as the least term, the common difference as 1, and the number of terms as 5, we find, by Rules I. and II., Arithmetic, page 267, the sum of the series to be 90 miles, the required distance.

36. By successive multiplications by 30, the given number is found to be equivalent to 235229409 in the decimal scale. Then, taking 4 from this, and dividing the remainder by 5, we get 47045881; which, by the

question, and by the nature of notation, is the sixth power of the required radix. Extracting the sixth root, therefore, we get 19, the answer.

Exer.

37. It is evident, that the sums paid to the boys, women, and men, for working the same number of hours, will be as 1, $1\frac{1}{3}$, and 2; or as 3, 4, and 6. Then, by multiplying the given hours by these numbers, we get $12\times6=72$, $9\times4=36$, and $8\times3=24$. Hence, the sums actually paid to each man, woman, and boy, must be as 72, 36, and 24; or as 6, 3, and 2. Again, the sums actually paid to all the men, women, and boys collectively, are obviously as 24, 10, and 5. Then, dividing these respectively by 6, 3, and 2, the numbers found above, we get 4, $3\frac{1}{3}$, and $2\frac{1}{2}$, which must be proportional to the number of the men, women, and boys. Lastly, as $4+3\frac{1}{3}+2\frac{1}{2} : 4 :: 59 : 24$, the number of the men: as $4+3\frac{1}{3}+2\frac{1}{2} : 3\frac{1}{3} :: 59 : 20$, the number of the women; and as $4+3\frac{1}{3}+2\frac{1}{2} : 2\frac{1}{2} :: 59 : 15$, the number of the boys.

38. The part of the property that remained after the first son's share, was $\frac{3}{4}$; and the part that remained after the second son's share, was $\frac{3}{4}$ of this, wanting £350; or $\frac{9}{16}$ of the whole, wanting £350. Again, $\frac{3}{4}$ of this is $\frac{27}{64}$ of the whole, wanting $\frac{3}{4}$ of £350; that is, $\frac{27}{64}$ of the whole, wanting £$262\frac{1}{2}$; and, therefore, what remained after the third son's share must have been $\frac{27}{64}$ of the whole, wanting £$1237\frac{1}{2}$, the sum of £$262\frac{1}{2}$ and £975: $\frac{3}{4}$ of this, again, is $\frac{81}{256}$ of the whole, wanting £$928\frac{1}{8}$; and, consequently, the widow's share, or what remained after all the son's shares, was $\frac{81}{256}$ of the whole, wanting £$2328\frac{1}{8}$, the sum of £$928\frac{1}{8}$ and £1400. But, by the question, this was equivalent to $\frac{1}{5}$ of the whole; and therefore the difference between $\frac{81}{256}$ of the whole, and $\frac{1}{5}$ of the whole, must have been £$2328\frac{1}{8}$. But, $\frac{81}{256}-\frac{1}{5}=\frac{149}{1280}$; and, therefore, as $\frac{149}{1280}$: 1, or as 149 : 1280 :: £$2328\frac{1}{8}$: £20000, the answer.

39. Since there are four yards in the greater for every three in the less, and since the less costs twice as many pence per yard as it contains yards, while the greater costs only as many pence per yard as there are yards, it

is evident, that the prices of a yard of the greater, and a yard of the less, will be as 4 to 6, or as 2 to 3. Hence, by multiplying these last numbers by 4 and 3, the numbers expressing the ratio of the quantities, we get 8 and 9, expressing the ratio of the entire prices. Now, the difference of these being 1, and the given difference of the prices being £31 - 0 - 2, it appears that the greater bale must have cost 8 times, and the less 9 times, £31 - 0 - 2. Hence, multiplying £31 - 0 - 2 by 8, reducing the product to pence, and extracting the square root, we get the number of yards contained in the greater; and three fourths of that result will be the number contained in the less.

Exer.

40. As $105 : 5 :: £1 : £\frac{1}{21}$, the true discount of £1 for a year, at 5 per cent. per annum. Also, as $104 : 4 :: £\frac{1}{2} : £\frac{1}{52}$, the true discount of $£\frac{1}{2}$ for a year, at 4 per cent. per annum; and as $106 : 6 :: £\frac{1}{2} : £\frac{3}{106}$, the true discount of $£\frac{1}{2}$ for a year at 6 per cent. per annum. Then, $\frac{1}{52} + \frac{3}{106} = \frac{131}{2756}$; and $\frac{1}{21} - \frac{131}{2756} = \frac{5}{57876}$. Then, as $\frac{5}{57876} : 1$, or as $5 : 57876 :: £1 : £11575 - 4$, the answer.

41. Subtracting the given ages severally from 21 years, we get 15, 13, 11, and 9 years. Then, by involution, $1{\cdot}045^{15} = 1{\cdot}93528244$; $1{\cdot}045^{13} = 1{\cdot}77219610$; $1{\cdot}045^{11} = 1{\cdot}62285305$; and $1{\cdot}045^{9} = 1{\cdot}48609514$; and dividing unity by these successively, we get ·51672044, ·56427164, ·61619874, and ·67290443, the respective present worths of £1 for 15, 13, 11, and 9 years. The sum of these is 2·37009525. Then,

As 2·37009525 : ·51672044 :: £10000 : £2180 - 3 - $4\frac{1}{4}$;
2·37009525 : ·56427164 :: £10000 : £2380 - 15 - $11\frac{1}{4}$;
2·37009525 : ·61619874 :: £10000 : £2599 - 17 - $9\frac{3}{4}$;
2·37009525 : ·67290443 :: £10000 : £2839 - 2 - $10\frac{3}{4}$.

42. Subtracting the given ages from 21 years, we get 15, 12, and 10 years. Then, $1{\cdot}04^{15} = 1{\cdot}80094351$, $1{\cdot}04^{12} = 1{\cdot}60103222$, and $1{\cdot}04^{10} = 1{\cdot}48024428$; and dividing 1 by these successively, we get for the present worths, ·55526450, ·62459705, and ·67556417. Now, the sum of the first and third of these, and of twice the second, is 2·48002277; and the sum of all three is

1·85543572; and if the whole property be supposed to be 1, and if all had survived the property left to the first would have been $\frac{\cdot 55526450}{2\cdot 48002277}$; and the product of this by $1\cdot 045^{15}$ would evidently be $\frac{1}{2\cdot 48002277}$; or, by division, ·40322210, the amount which he would receive on coming to age. It would be shown, in a similar manner, that the sum which he would receive on coming to age, after the death of one of the twins, would be $\frac{1}{1\cdot 85542572}$; or, by division, ·53895987. Now, the difference of these is ·13573777; and this corresponding to £1000, we have ·13573777 : 1 :: £1000 : £7367 - 2 - $11\frac{1}{4}$, the answer. It is evident that the share of the second or last might have been employed with equal facility, and that the result would have been the same.

Exer.

43. By discount, as £$101\frac{3}{4}$: £100 :: £1 : £$\frac{80}{81}$, the present worth of £1 due at 4 months, at 5 per cent. per annum; as £$103\frac{3}{4}$: £100 :: £1 : £$\frac{80}{83}$, its present worth at 9 months; as £105 : £100 :: £1 : £$\frac{20}{21}$, its present worth at 12 months; and as £$108\frac{1}{3}$: £100 :: £1 : £$\frac{12}{13}$, its present worth at 20 months. The sum of these is the present worth of £4 due, as mentioned in the question; therefore, as £$\frac{80}{81}$ + £$\frac{80}{83}$ + £$\frac{20}{21}$ + £$\frac{12}{13}$: £4 :: £750 : £$784\frac{9781924}{13210009}$, the answer.

44. $1\frac{1}{10} = 1\cdot 1$, and $1\cdot 1^{20} = 6\cdot 72749995$; the product of which with £5000 is £33637·49975, the sum which the merchant would have been worth, had there been no expenditure. Then, to find the amount of an annuity of £100 for the same time, we divide $6\cdot 72749995 - 1$ by ·1, and multiply the quotient, 57·2749995, by £100; the product is £5727·49995; and taking this from £33637·49975, we get £27909·99980, or £27910, very nearly, which is the answer.

45. As 12 months : 11 months :: £5 : £$4\frac{7}{12}$; as 12 months : 19 months :: £5 : £$7\frac{11}{12}$; and as 12 months : 30 months :: £5 : £$12\frac{1}{2}$. Then, as £$104\frac{7}{12}$: £100 :: £1 : £$\frac{240}{287}$, the amount of the debt which would be discharged by the payment of £1 at the end of 11 months. Again, as £$107\frac{11}{12}$: £100 :: £2 : £$1\frac{221}{239}$, the amount of the debt which would be discharged by the payment of £2 at the end of 19 months. In like manner, as

£112½ : £100 :: £2 : £1$\frac{7}{9}$, the part of the debt that would be discharged by the payment of £2 at the end of 30 months. The sum of these results is $4\frac{343580}{585081}$. Then, as $4\frac{343580}{585081}$: 1 :: £500 : £108$\frac{669717}{670976}$, the first payment, which is to be doubled to find each of the others.

Exer.

46. By reducing the given periods to seconds, and performing the division in the usual manner, the quotients are found to be 4, 6, 1, 1, 2, 1, 4, &c. Then,

4	6	1	1	2	1	4	&c.
$\frac{1}{4}$	$\frac{6}{25}$	$\frac{7}{29}$	$\frac{13}{54}$	$\frac{33}{137}$	$\frac{46}{191}$	$\frac{217}{901}$,	&c.

47. By reducing the given periods to seconds, and dividing in the usual manner, we find the quotients to be 1, 1, 1, 1, 2, 29, 2, 30, &c. Then,

1	1	1	1	2	29	2	30	&c.
$\frac{1}{1}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{5}$	$\frac{8}{13}$	$\frac{235}{382}$	$\frac{478}{777}$	$\frac{14575}{23692}$,	&c.

48. 35 lbs. + 20 lbs. = 55lbs.; and 55 lbs. at 7*s.* 4*d.* = £20 - 3 - 4, the value of the whole mixture. Also, 35 lbs. at 1*s.* 10*d.* per lb. = £3 - 4 - 2; and £20 - 3 - 4 + £3 - 4 - 2 = £23 - 7 - 6, the sum which the whole would have cost at the price of the better kind, and which is therefore to be divided by 55 to find the price per lb. of the better; and if 1*s.* 10*d.* be taken from that price, the remainder will be the price of the other.

The price of the better kind might also be found by adding to 7*s.* 4*d.* $\frac{35}{55}$, or $\frac{7}{11}$, of 1*s.* 10*d.*

49. The yearly interest of £7000, at 6 per cent. is £420; and £420 − £240 = £180. Then (Arithmetic, Rule I., page 281), find the amount of an annuity of £180 for 16 years, at 6 per cent. per annum, and add to it the original debt of £7000, to find the answer.

The answer might also be found by taking the amount of an annuity of £240 for 16 years, from the amount of £7000 at compound interest for the same time.

50. In each of the last 14 days the boy reads 36 lines more than in each corresponding day of the first 14; and, therefore, 36 × 14 = 504, the number of lines read in the fifteenth, sixteenth, seventeenth, and eighteenth days. The half of this, 252, is the number of lines read in the two *mean* days; and the product of this by 16, half the number of terms, is the answer.

Exer.

51. A is 31 (308 − 277) yards farther from the gate than B, and goes $\frac{1}{3}$ of a yard per second more than B. Therefore, as $\frac{1}{3}$ yard : 31 yards :: 1 second : 1 minute 33 seconds, the answer.

Again, $2\frac{1}{3} + 2 = 4\frac{1}{3}$, and $277 + 308 = 585$. Then, as $4\frac{1}{3}$ yards : 585 yards :: 1 second : 2 minutes 15 seconds, the time in which they will meet, when they will also be equally distant from the gate.

52. It will appear, on a little consideration, that A, the swifter of them, must have travelled over the entire distance between them, and must have passed the station of B, as far as B wants of having reached the station of A; and, therefore, the spaces travelled over by both, must be together double of the entire distance. Hence, $2\frac{1}{3} + 2 = 4\frac{1}{3}$; $308 + 277 = 585$; and $585 \times 2 = 1170$. Then, as $4\frac{1}{3}$ yards : 1170 yards :: 1 second : 4 minutes 30 seconds, the answer.

53. The amount of £1100 for 11 years, at 6 per cent. per annum, compound interest, is £2088·1289; and the amount of an annuity of £1 for the same time, and at the same rate, is 14·971642. Then, to find the answer, divide the former amount by the latter.

54. £1 - 7 - 6 − 10*s.* = 17*s.* 6*d.*; and 84 acres at 17*s.* 6*d.* = £73 - 10. Then, the rest of the work is the same as to find the present value of a reversion of £73 - 10, at 6 per cent. per annum, to commence after 18 years, and to continue 30 years. To do this by Rule V. Arithmetic, page 285, we have $1{\cdot}06^{30} = 5{\cdot}743491$, and $1{\cdot}06^{48} = 16{\cdot}393872$. Taking 1 from the first of these, and dividing the remainder by the second, we get ·2893454; which, again, is to be divided by ·06; and the quotient, 4·82242, is to be multiplied by 73·5 (for £73 - 10), to find the answer.

55. By Table IV., the values at 4 per cent. of two lives of 45 and 40 are 13·901 and 15·135; the sum of which is 29·036. Also, by Table V., the value of £1 on the joint continuance of the same lives is 11·656; the difference between which and the sum above obtained is 17·38. Then, multiplying 13·901 by 300, we get

Then, as $£\frac{102}{9800}$: $£\frac{106}{9700}$; or, by dividing the numerators by 2, and the denominators by 100, as $£\frac{51}{98}$: $£\frac{53}{97}$; or, by reducing the fractions to a common denominator, and using only the numerators, as £4947 : £5194:: £10000 : $£10499\frac{1447}{4947}$, the answer.

Exer.

64. By extracting the square root of 5, and dividing by 2, we get 1·118034; and by taking ·5 from this we obtain ·618034. Then employing this and unity, we get, by the usual divisions, the quotients 1, 1, 1, 1, 1, &c.; and the work for finding the converging fractions will stand thus:—

1	1	1	1	1	1	1	1	&c.
	$\frac{1}{1}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{5}$	$\frac{5}{8}$	$\frac{8}{13}$	$\frac{13}{21}$	&c.

Again, by taking 1·118034 from 1·5 ($=\frac{3}{2}$), we get ·381966: and by using this and unity, and by the usual divisions, we find for quotients 2, 1, 1, 1, 1, &c. The rest of the work will then be as follows:—

2	1	1	1	1	1	1	
	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{5}$	$\frac{3}{8}$	$\frac{5}{13}$	$\frac{8}{21}$	&c.

APPENDIX.

MENSURATION OF SURFACES.

(Arithmetic, page 331.)

Exer. 1. 1276 links × 943 links = 1203268 square links, or 12 acres 0 roods 5 perches.

2. Here, 66 feet 8 inches = 66·6̇ feet, and 937 feet 6 inches = 937·5 feet. The product of these is 62500 square feet, which is divided by 9 to reduce it to square yards; and the price of these is found at 8½*d.* per yard. The work is follows:—

66·66̇
937·5
333333̇
4666666̇
200000
600000
62499·9999̇, or
9) 62500 square feet.
6944·4444̇ square yards, at 8½*d.*

6*d.* = £$\frac{1}{40}$	173·6111̇
2*d.* = $\frac{1}{3}$ of 6*d.* ...	57·8704
½*d.* = $\frac{1}{4}$ of 2*d.* ...	14·4676
	£245·9491, or
	£245 - 18 - 11$\frac{3}{4}$.

3. Here, the product of the given dimensions is found to be 714 feet 2′ 10″; or, by division by 9, 79 yards 3 feet 2′ 10″; the price of which, at 9¾*d.* per yard, is the answer. The work is as follows:—

	Feet.	*Inches.*	
	70	10	
	10	1	
	708	4	
	5	10	10
9)	714 feet	2′	10″

79 yards 3 feet 2′ 10″ at $9\frac{3}{4}d.$

$6d.=\frac{1}{2}s.$....	39	6
$3d.=\frac{1}{2}$ of $6d.$	19	9
$\frac{3}{4}d.=\frac{1}{4}$ of $3d.$	4	$11\frac{1}{4}$
3 f.$=\frac{1}{3}$ yd....	0	$3\frac{1}{4}$
For 2′ 10″...		$\frac{1}{4}$
	64	$5\frac{3}{4}$, or

£3 - 4 - $5\frac{3}{4}$.

4. $98 \times 81 = 7938$ square yards; which, by reduction to acres, will give the answer.

5.

Feet.	*Inches.*	
9	8	
	$8\frac{1}{2}$	
6	5	4
	4	10
6	10′	2″

6.

Feet.	*Inches.*	
3	10	
3	10	
11	6	
3	2	4
14	8′	4′

7. $1324 \times 859 = 1137316$; $1137316 \div 2 = 568658$ square links, or 5 acres 2 roods 30 perches, nearly.

8.

	Feet.	*Inches.*	
	21	7	
	17	10	
	366	11	
	17	11	10
2)	384	10	10
	192	5′	5″

Exer.

9. Here half the sum of the given sides is 1160 links, and the three remainders are 198 links, 552 links, and 440 links. The continual product of these is 52752902400, the square root of which is 229680 square links; or, by division by 100000, and by reduction, 2 acres 1 rood $7\frac{1}{2}$ perches, nearly.

10. The half sum is 21, and the three remainders are 8, 7, and 6. The continued product of these is 7056; the square root of which is 84, the answer.

11. Here, the half sum of the given sides in inches is 88, and the three remainders are 44, 33, and 11. The continual product of these is 1405536; the square root of which is 1185·55304 square inches; or, by division by 144, 8·233 square feet.

12. $33+28=61$; $61\times11=671$; and $671\div2=335\frac{1}{2}$ inches, the answer.

13. $75+33=108$; $108\times20=2160$; and $2160\div2=1080$ square feet; or, by division by 9, 120 square yards.

14. 882 links + 773 links = 1655 links; 1655 links × 1756 links = 2906180 square links; the half of which is 1453090 square links, or 14 acres 2 roods 5 perches, nearly.

15. In the triangle ABD, the half sum is 24, and the remainders are 3, 6, and 15. The continual product of these is 6480; the square root of which is 80·49845, the area of ABD.

Again, in the triangle BCD, the half sum is 24, and the remainders are 3, 12, and 9. The continual product of these is 7776, the square root of which is 88·18163, the area of CBD.

Lastly, the sum of the two areas above found is 168·68008, the area of the trapezium.

16. $27+18=45$. Then, $45\times58\div2=1305$ square perches, the area of the trapezium AEDC.

Again, $46\times20\div2=460$ square perches, the area of the triangle ABC.

Lastly, the sum of the areas is 1765 square perches; or, by reduction, 11 acres 0 roods 5 perches, the area of the whole figure.

Exer.

17. First, in the triangle ABC, half the sum of the sides is 8, and the remainders are 2, 5, 1, the continual product of which is 80; and the square root of that product is 8·944272, the area of ABC.

Secondly, in the triangle CDE, the half sum is 8·5, and the three remainders are 4·5, 3·5, and ·5. The continued product of these is 66·9375; the square root of which is 8·181534, the area of CDE.

Thirdly, in the triangle ACE, the half sum is 9·5, and the remainders are 5·5, 2·5, and 1·5. The continual product of these is 195·9375; the square root of which is 13·997768, the area of ACE.

Lastly, the sum of these three areas is 31·123574, the area of the whole polygon.

18. $5^2 = 25$, and $2{\cdot}5980762 \times 25 = 64{\cdot}951905$, the area.

19. $60^2 = 3600$, and $4{\cdot}8284272 \times 3600 = 17382{\cdot}33792$ square yards. Then, by dividing this by $30\frac{1}{4}$, we get 574 perches 19 yards, nearly; or, by farther division by 40 and 4, 3 acres 2 roods 14 perches 19 yards.

20. $3{\cdot}141593 \times 13 = 40{\cdot}840709$; or, as 113 : 355 :: 13 : 40·840708, nearly; or, as 7 : 22 :: 13 : 40·857.

21. 2 feet 9 inches = 33 inches. Then, $3{\cdot}141593 \times 33 = 103{\cdot}672569$; or, as 113 : 355 :: 33 : 103·672566; or, as 7 : 22 :: 33 : 103·7; any of which becomes, by reduction, 8 feet 7 inches, with a fraction.

22. 12 feet 5 inches = 149 inches. Then, as 355 : 113 :: 149 : 47·428, &c., inches; or, as 22 : 7 :: 149 : 47·409; or $149 \div 3{\cdot}1416 = 47{\cdot}4$, &c.

23. $31^2 = 961$; $\cdot7854 \times 961 = 754{\cdot}7694$ square yards, the answer.

24. Here, the diameter is 15 perches; and $15^2 = 225$. Then, $\cdot7854 \times 225 = 176{\cdot}7150$ square perches; or, by reduction, 1 acre 0 roods 16·715 perches.

25. $56{\cdot}5^2 = 3192{\cdot}25$. Then, $3192{\cdot}25 \times \cdot07958 = 254{\cdot}0392550$ square perches, or 1 acre 2 roods 14 perches.

Exer.

26. 5 feet 8 inches = 68 inches, and $68^2 = 4624$. Then, $\cdot 7854 \times 4624 = 3631\cdot 6896$ square inches, which is to be reduced to square feet, to find the answer.

27. By reducing the axes to inches, we get 401 and 243. Then, $401 \times 243 = 97443$, and $97443 \times \cdot 7854 = 76531\cdot 7322$ square inches; which, by reduction, becomes 59 yards 0 feet $67\cdot 7322$ inches.

MENSURATION OF BODIES, OR SOLIDS.

(Arithmetic, page 336.)

Exer. 1. By reducing the dimensions to inches, we get 192, 126, and 100, the continual product of which is 2419200; and this is to be divided by $277\cdot 274$ to find the answer.

2. $2^2 = 4$. Then (by Rule III., Arithmetic, page 331), $\cdot 4330127 \times 4 = 1\cdot 7320508$, the area of the base; and $1\cdot 7320508 \times 14 = 24\cdot 2487112$, the answer.

3. $24 \times 18\frac{1}{2} = 444$; and $444 \times 10\frac{7}{12} = 4699$ cubic feet, the answer.

4. The dimensions in inches are 37 and 225. Then, $37^2 = 1369$; $1369 \times \cdot 7854 = 1075\cdot 2126$, the area of the base; and $1075\cdot 2126 \times 225 = 241922\cdot 8350$, the content in cubic inches, which is to be divided by 1728 to reduce it to feet.

5. This exercise would be wrought very easily by multiplying the feet and inches, without reducing them. It may also be easily wrought thus: $4\frac{2}{3} \times 3\frac{1}{4}$, or $\frac{14}{3} \times \frac{13}{4} = \frac{91}{6}$; and $\frac{91}{6} \times 2\frac{1}{2}$, or $\frac{91}{6} \times \frac{5}{2} = \frac{455}{12} = 37\frac{11}{12}$ feet.

6. In this exercise, 22 feet 7 inches $= 22\cdot 58\dot{3}$. Then, in multiplying this by 1 foot 5 inches, the method of aliquot parts is employed; and in a similar manner the result is multiplied by $6\frac{1}{2}$ inches, as below. The answer might also be found by duodecimals, or by reducing the dimensions to inches, and dividing their continued product by 1728.

	Feet.
	22·583333
	1 foot 5 inches.
	22·583333
4 inches = $\frac{1}{3}$ foot ...	7·527777
1 inch = $\frac{1}{4}$ of 4 in.	1·881944
	31·993055
6 inches = $\frac{1}{2}$ foot ...	15·996527
$\frac{1}{2}$ inch = $\frac{1}{12}$ of 6 in.	1·333044
	17·329571

Exer.

7. 10 inches=$\frac{5}{6}$ foot; then, $\frac{5}{6}\times\frac{5}{6}=\frac{25}{36}$ square foot. Again, one third of the height is 3 feet 3 inches, or $3\frac{1}{4}$ feet and $\frac{25}{36}\times 3\frac{1}{4}$, or $\frac{25}{36}\times\frac{13}{4}=\frac{325}{144}$ cubic feet, or by division by 12 and 12, 2 feet 3′ 1″, the answer. This might also be wrought by duodecimals, or by reducing 9 feet 9 inches to inches.

8. 37 feet 8 inches=452 inches, and 79 feet 9 inches =957 inches. Then, 452^2=204304, and 204304 × ·7854=160460·3616, the area of the base. Again, 160460·3616 × 957=153560566·0512, one third of which is 51186855·3504, the content in cubic inches; and dividing this by 1728 (=12 × 12 × 12) we get 29622·0228 cubic feet, nearly.

9. 720^2=518400, the area of the base. Then, 518400 × 477=247276800, one third of which is 82425600, the content in cubic feet, which is to be divided by 27 to reduce it to cubic yards.

10. 9^2=81, and ·7854 × 81=63·6174, square inches, the area of the base. Then, 63·6174 × 17=1081·4958, one third of which is the answer in cubic inches.

11. $1·75^2$=3·0625, and ·7854 × 3·0625=2·4052875, the area of the mouth in square inches; and since one third of the depth is 1, the content in cubic inches will also be 2·4052875. Then, the answer will be obtained by dividing 277·274 by the content above found.

Exer.

12. $27\times16=432$; $27-16=11$; $11^2\div3=40\frac{1}{3}$; and $432+40\frac{1}{3}=472\frac{1}{3}$, or 472·3, the area of the mean base in square inches; and if this be divided by 144, the quotient is 3·2800925, the area in square feet. Then, the answer will be obtained by multiplying this area by 18, and taking aliquot parts for 8 inches.

13. $2\frac{1}{2}=2{\cdot}5$, and $2{\cdot}5\times1=2{\cdot}5$; also, $2{\cdot}5-1=1{\cdot}5$, the difference of the diameters; $1{\cdot}5^2\div3={\cdot}75$; and $2{\cdot}5+{\cdot}75=3{\cdot}25$, the square of a mean diameter. Then, $\cdot7854\times3{\cdot}25=2{\cdot}55255$, and the product of this by the depth is the answer.

14. $12^3=1728$, $15^3=3375$, and $21^3=9261$. Then, to find the answers, multiply these successively by ·5236.

15. $38^2=1444$, and $1444\times48=69312$; the product of which by ·5236 is the answer.

16. By multiplying 2 feet 8 inches by 3 feet 9 inches, and taking half the product, we get 5 feet for the area of each slant side; and 4 times this, or 20 feet, is the area of the four slant sides. Then, multiplying 2 feet 8 inches by itself, we get, for the area of the base 7 feet, 1′ 4″; the sum of which, and of the former result, is the answer.

17. 2 feet 11 inches=35 inches, and 4 feet 7 inches=55 inches. Then, $3{\cdot}1416\times35=109{\cdot}956$, the circumference of the base. Multiplying this by 55, the slant height, and taking half the product, we get for the area of the curve surface, 3023·79 square inches. Again, $109{\cdot}956\times35\div4=962{\cdot}115$ square inches, the area of the base. By adding these areas together, we get, for the entire area, 3985·905 square inches; which is to be divided by 144 to find the area in square feet.

The answer might be found rather more easily by adding together the slant height and the radius of the base, multiplying the sum by the circumference of the base, and taking half the product.

18. $7912^2=62599744$; which multiply by 3·1416 to find the answer.

19. $12^2=144$, $15^2=225$, and $21^2=441$; and, to find the answers, multiply these successively by 3·1416.

Exer.

20. Here, the quarter girt is 20 inches, and the length is 44 feet 4 inches, or 532 inches. Then, $20^2 = 400$, and $400 \times 532 = 212800$, the content in cubic inches; which is to be divided by 1728 to reduce it to cubic feet.

The answer would also be easily found by multiplying 1 foot 8 inches by itself without reduction, and the product by 44 feet 4 inches. In this way, the first product would be found to be 2 feet 9′ 4″, and the answer 123 feet 1′ 9″ 4‴.

21. Here, the quarter girt is 17·5 inches, the square of which is 306·25 square inches, or 2·126736 square feet, the mean area. Then, to find the answer according to the common method, multiply this by 3·25, the length.

To find the answer more nearly correct, square the girt, 70 inches, and multiply the result, 4900, by ·08; the product is 392 square inches, or 2·722 square feet, the area of the mean section. Then, the product of this by 32·5 will be 88·465; and taking from this one 190th of itself, or a little more than half a foot, there will remain about 87·9 cubic feet, the answer.

22. By multiplying 2 feet 7 inches by 1 foot 9 inches, we get 4 feet 6′ 3″, the area of the middle section. Then, by multiplying this by the length, 31 feet 4 inches, we obtain 141 feet 7′ 10″, the answer.

The answer may also be found by multiplying the breadth, 31 inches, by the depth, 21 inches; and the product, 651 square inches, by 376, the length in inches. Then, the product, 244776 cubic inches, will be reduced to feet by dividing it by 1728.

As a third method, we might multiply the breadth, $2{\cdot}58\dot{3}$ feet, by the depth, 1·75 feet; and the product, $4{\cdot}52083$ square feet by $31{\cdot}\dot{3}$.

23. The sum of the girts is 31 feet 8 inches; one fourth of which is 7 feet 11 inches, or 95 inches, the mean girt. Then, by the common method, $95 \div 4 = 23{\cdot}75$, and $23{\cdot}75^2 = 564{\cdot}0625$ square inches, or 3·9171 square feet. The answer will be found by multiplying this by 21 feet, and taking aliquot parts for 5 inches.

To find the answer by the second method, $95^2=9025$; $9025\times\cdot08=722$ square inches, or 5·01388 square feet; the product of which by 21 feet 5 inches is 107·38; from which a little more than half a foot is to be taken, to find the answer.

Exer.

24. $25^2=625$; $34^2=1156$; $1156\times2=2312$; and $625+2312=2937$. Again, $34-25=9$; $9^2=81$; $81\times\cdot4=32\cdot4$; and $2937-32\cdot4=2904\cdot6$; the product of which by 43 is 124897·8. Then, multiplying this by 0·0009442, we get 117·93, or 118 gallons, nearly.

25. $160\div15=10\frac{2}{3}$ perches.

26. Since the product of the square of the radius by 3·1416 must be equal to 160 perches, it follows that the radius, or the length of the cord, will be found by dividing 160 by 3·1416, and extracting the square root of 50·92946, the quotient.

27. $2\frac{1}{4}$ feet $\times3=6\frac{3}{4}$; also $3-2\frac{1}{4}=\frac{3}{4}$; the square of which is $\frac{9}{16}$; and $\frac{9}{16}\div3=\frac{3}{16}$. Then, $6\frac{3}{4}+\frac{3}{16}=6\frac{15}{16}$, the square of the mean diameter. Let ·7854 be multiplied by the number last found, and the product, 5·4487125, is the area of the mean section. Multiply this by 28·5, the length, and the product is 155·28830625. Lastly, let this be multiplied by 2568, and the product is 398780 ounces; which, by reduction, will become the answer.

28. Here, the square of the diameter multiplied by ·7854 must be equal to 144 square inches; and, therefore, $144\div\cdot7854=183\cdot346065$; the square root of which is the answer.

29. Here, the product of the cube of the diameter by ·5236 must be equal to 1728 cubic inches; and, therefore, $1728\div\cdot5236=3300\cdot229182$; the cube root of which is the answer.

30. $50^2\times\cdot7854=1963\cdot5$ square inches, the area of the base. Then, 50 cubic feet=86400 cubic inches; $86400\times3=259200$; and $259200\div1963\cdot5=132\cdot00917$ inches, or 11 feet 0·00917 inches, the answer.

Exer.

31. $22\frac{1}{2} \times 16\frac{2}{3} = 375$ square feet, the area of the floor. Then, since 27 inches = 2 feet 3 inches = $2\frac{1}{4}$ feet, we have $375 \div 2\frac{1}{4}$, or $1500 \div 9 = 166\frac{2}{3}$ feet = 55 yards $1\frac{2}{3}$ feet = 55 yards 1 foot 8 inches, the length required.

32. Since 63 feet = 21 yards, and since a mile = 1760 yards, and consequently 100 miles = 176000 yards, we have $176000 \times 21 = 3696000$ square yards, the area. Now, since a square perch = $30\frac{1}{4}$ square yards, we have $30\frac{1}{4} \times 160 = 4840$, the square yards in an acre. Lastly, by dividing 3696000 by 4840, we get $763\frac{7}{11}$ acres, the answer.

33. Here, the exterior diameter is plainly $3\frac{1}{2}$, or 3·5 inches. Then, $3{\cdot}5^2 = 12{\cdot}25$ and $3^2 = 9$, the difference of which is 3·25. Multiplying this by ·7854, we get 2·55255 square inches, the area of the end of the tube: and multiplying this by 100, the length, and dividing the product by 144, we obtain 1·7726 cubic feet, the solid content of the metal of the pipe. Lastly, by multiplying this by 7000, we get 12408 ounces; or, by reduction, 6 cwt. 3 qrs. $19\frac{1}{2}$ lbs., nearly.

Again, to find the content of the interior of the tube, we multiply ·7854 by 9, and the product by 100; then, dividing the result by 144, we get 4·90875 cubic feet, the content. This is then multiplied by 1000, and the product is $4908\frac{3}{4}$ ounces, or, by reduction, 2 cwt. 2 qrs. 27 lbs., nearly.

34. $27^2 + 37^2 \times 2 = 729 + 1369 \times 2 = 3467$: also $37 - 27 = 10$, and $10^2 \times \frac{4}{10} = 40$. Then, $3467 - 40 = 3427$, and $3427 \times 45 = 154215$; the product of which by 0·0009442, is 145·6 gallons, nearly. Multiplying this by 10, because each gallon weighs 10 lbs., we get 1456 lbs., or, by reduction, 13 cwt.

35. $0{\cdot}7854 \times 3^2 = 7{\cdot}0686$, and $7{\cdot}0686 \times 5 = 35{\cdot}343$.

RESOLUTION OF EQUATIONS.

(Arithmetic, page 357.)

Exer. 1. Here, by taking $x=0$ and $x=10$, we get 25 and 5, instead of 0; and, these errors having the same signs, it follows by the nature of equations, that either no root or both roots lie between 0 and 10. Again, by taking $x=5$, we get -10; which differing in sign from 25 and 5, before obtained, one root must lie between 5 and 10, and the other between 0 and 5; and it is easily found by farther trials, that one of them lies between 9 and 10, and the other between 2 and 3. The work for finding the former is as follows :—

-12	$25 \,(\, 9{\cdot}3166248$
9	-27
-3	-200
9	189
60	-1100
3	661
63	-43900
3	39756
660	-414400
1	397956
661	-16444
1	13266
6620	-3178
6	2653
6626	-525
6	530
66320	
6	
66326	
6	
66332	

(2.)

10	−1700	10000 (0·625670
6	96	−9624
16	−1604	376000
6	132	−293272
22	−147200	827280
6	564	−729620
280	−146636	97660
2	568	−87456
282	−146068	10204
2	144	−10202
284	−145924	2
2	144	
286	−145780	
	2	
	−14576	
	2	
	−14574	

(3.)

1	−17	10 (−4·8924879
−4	12	20
−3	−5	30000
−4	28	−25952
−7	2300	4048000
−4	944	−3936069
−110	3244	1119310
−8	1008	−899672
−118	425200	219638
−8	12141	−180064
−126	437341	39574
−8	12222	−36017
−1340	449563	3557
−9	273	−3151
−1349	449836	406
−9	273	−405
−1358	450109	
−9	5	
−1367	45016	
	5	
	45021	

By the nature of eqnations, this last root would have been found much more easily, by adding the first and second roots together, and subtracting their sum from -1, the co-efficient of the second term with its sign changed.

Exer.

3. Here it is easily found, that one root lies betweer 3 and 4, and it will be computed thus :—

0	-7	-9 (3·14092334
3	9	6
3	2	-3000
3	18	2091
6	2000	-909000
3	91	888144
90	2091	-2085600
1	92	2032857
91	218300	-52743
1	3736	45192
92	222036	-7551
1	3752	6779
930	2257880	-772
4	85	678
934	225873	-94
4	85	90
938	225958	
4		
942		

To find the remaining roots, or their nature, we must divide the first member of the given equation by $x-3{\cdot}1409233$, and those roots will be the roots of the equation found by putting the quotient equal to nothing. The division by the method of detached co-efficients, is as follows :—

0	-7	-9 (3·140923
3·1409233	9·8653992	8·9999991
3·1409233	2·8653992	$-0{\cdot}0000009$

Hence, the equation to be resolved is

$$x^2+3{\cdot}1409233x+2{\cdot}8653992=0;$$

the roots of which are imaginary, since 2·8653992 is greater than the square of the half of 3·1409233.

Exer.

4. Here it will be found, that one of the roots is between 6 and 7, another between 1 and 2, a third between 0 and −1, and the fourth between −3 and −4. The work for finding them is as follows :—

(1.)	−4	−20	36	11 (6·236068
	6	12	−48	−72
	2	−8	−12	−610000
	6	48	240	507216
	8	40	228000	−10278400
	6	84	25608	8524134
	14	12400	253608	−1754266
	6	404	26424	1734546
	200	12804	2800320	−19720
	2	408	41058	17395
	202	13212	2841378	−2325
	2	412	41244	2319
	204	13624	2882622	
	2	62	829	
	206	13686	289091	
	2	62	829	
	208	13748	289920	
		62		
		13810		

(2.)	−4	−20	36	11 (1·763932
	1	−3	−23	13
	−3	−23	13	240000
	1	−2	−25	−208999
	−2	−25	−12000	31001000
	1	−1	−17857	−29040768
	−1	−2600	−29857	1960232
	1	49	−17171	−1494957
	00	−2551	−4702800	465275
	7	98	−137328	−449280
	7	−2453	−4840128	15995
	7	147	−136296	−14982
	14	−23060	−4976424	1013
	7	172	−676	−999
	21	−22888	−498319	14
	7	172	−676	
	28	−22716	−498995	
		172	−20	
		−22544	−49920	
			−20	
			−49940	

Here, at the commencement of the work for the third figure of the root, only three ciphers are annexed in the fourth column, two in the third, &c.: these being sufficient for giving the root true for five or six places of decimals.

(3.)

−40	−2000	36000	110000	(−0·26794919
−2	84	3832	−79664	
−42	−1916	39832	303360000	
−2	88	3656	−267072624	
−44	−1828	43488000	−36287376	
−2	92	1024104	31943506	
−46	−173600	44512104	−4343870	
−2	2916	1006392	4118688	
−480	−170684	45518496	−225182	
−6	2952	11508	183112	
−486	−167732	4563358	−42070	
−6	2988	11487	41201	
−492	−164744	4574845	−869	
−6	3	147	458	
−498	−1644	457632	−411	
−6	3	147	411	
−504	−1641	457779		
	3	1		
	−1638	45778		
		1		
		45779		

(4.)	−4	−20	36	11 (−3·7320508
	−3	21	−3	−99
	−7	1	33	−880000
	−3	30	−93	820281
	−10	31	−60000	−5971900
	−3	39	−57183	5583150
	−13	7000	−117183	−388750
	−3	1169	−65709	379102
	−160	8169	−1828920	−9648
	−7	1218	−32130	9488
	−167	9387	−1861050	−160
	−7	1267	−32298	152
	−174	10654	−1893348	
	−7	56	−216	
	−181	10710	−189551	
	−7	56	−216	
	−188	10766	−189767	
		56		
		10822		

After finding the first and second roots, we might find the third and fourth more easily than has been done above, by attaching the first and second separately to x, with their signs changed, and finding the product, $(x-6{\cdot}236068)\ (x-1{\cdot}763932)$; which, by actual multiplication, turns out to be $x^2-8x+11$. Then, by dividing the first member of the given equation by this, and putting the quotient=0, we get $x^2+4x+1=0$; the roots of which are the third and fourth roots of the given equation.

Exer.

5. It is found by trial, that one root of this equation lies between 5 and 6, and another between -1 and -2; and these are computed in the following manner:—

(1.)	6	−22	−198	−243 (5·60555127, or
	5	55	165	−165 5·6055513, nearly.
	11	33	−33	−4080000
	5	80	565	4034256
	16	113	532000	−45744000
	5	105	140376	41193975
	21	21800	672376	−4550025
	5	1596	150168	4126745
	260	23396	82254400	−423280
	6	1632	13355	412750
	266	25028	8238795	−10530
	6	1668	13360	8255
	272	266960	8252155	−2275
	6	1	134	1651
	278	2671	825349	−624
	6	1	134	578
	284	2672	825483	
			1	
			82550	

(2.) 6	−22	−198	−243	(−1·60555127, or
−1	−5	27	171	−1·6055513, nearly.
5	−27	−171	−720000	
−1	−4	31	714576	
4	−31	−140000	−5424000	
−1	−3	20904	4886340	
3	−3400	−119096	−537660	
−1	−84	21192	487660	
20	−3484	−9790400	−50000	
−6	−48	1772	48757	
14	−3532	−977268	−1243	
−6	−12	1772	975	
8	−35440	−975496	−268	
−6		17	195	
2		−97532	−73	
−6		17	68	
−4		−97515		

Now, the product of $x-5{\cdot}6055513$ and $x+1{\cdot}6055513$ is x^2-4x-9; and, by dividing the first member of the given equation by this, we get for the quadratic equation which has the other two roots, $x^2+10x+27=0$. The roots of this, however, are imaginary.

Ex.

6. Here, the equation is

$$x^5+x^4+x^3+x^2+x-100=0,$$

and it is readily found, that the value of x is between 2 and 3. The work then proceeds thus:—

1	1	1	1	−100 (2·2396431
2	6	14	30	62
3	7	15	31	−3800000
2	10	34	98	3064992
5	17	49	1290000	−73500800
2	14	62	242496	55195902
7	31	111000	1532496	−18304898
2	18	10248	263904	17075412
9	4900	121248	17964000	−1229486
2	224	10704	434634	1146972
110	5124	131952	18398634	−82514
2	228	11168	439944	76504
112	5352	143120	18838578	−6010
2	232	1758	13410	5738
114	5584	144878	1897268	−272
2	236	1770	13455	191
116	5820	146648	1910723	−81
2	4	1782	90	
118	586	148430	191162	
2	4	5	90	
120	590	1490	191252	
	4	5	1	
	594	1495	19126	
		5	1	
		1500	19127	

The other roots are imaginary.

HINTS FOR TEACHERS.

WHILE, by means of this Key, in connexion with the Arithmetic, much time will be saved to the teacher, and while the Arithmetic contains a great number of exercises, still it may often be desirable that other questions should be proposed to the pupil, if it can be done without encroaching too much on the time of the master. The following hints may be useful with regard to this object.

From the exercises in the Arithmetic in addition, both simple and compound, the teacher may form many others, by causing the pupil to combine two or more of those given there into one; and in every such case, the answer will evidently be the sum of the corresponding answers given at the end, in pages 347 and 348. In this way, he may form, with ease, and in a short time, the answers of a great number of exercises, and may register them for future use—perhaps in the blank leaves of the Key. The following, formed from the first six exercises in simple addition, will serve as a specimen:—

1 + 2 = 227898
1 + 3 = 1530494
1 + 4 = 1405013
1 + 5 = 21246647
1 + 6 = 925943

2 + 3 = 1715282
2 + 4 = 1589801
2 + 5 = 21431435
2 + 6 = 1110731

3 + 4 = 2892397
3 + 5 = 22734031
3 + 6 = 2413327

4 + 5 = 22608550
4 + 6 = 22877846

5 + 6 = 22129480

1 + 2 + 3 = 1736837
1 + 2 + 4 = 1611356
1 + 2 + 5 = 21452990

1 + 2 + 6 = 1132286

1 + 3 + 4 = 2913952
1 + 3 + 5 = 22755586
1 + 3 + 6 = 2434882

1 + 4 + 5 = 22630105
1 + 4 + 6 = 2309401

1 + 5 + 6 = 22151035

2 + 3 + 4 = 3098740
2 + 3 + 5 = 22940374
2 + 3 + 6 = 2619670

2 + 4 + 5 = 22814893
2 + 4 + 6 = 2494189

2 + 5 + 6 = 22335823

3 + 4 + 5 = 24117489
3 + 4 + 6 = 3796785

3 + 5 + 6 = 23638419
4 + 5 + 6 = 23512938

In the first of the foregoing, the numbers (3789, &c.) in Exercise 1, page 18, are to be written down, and below them the numbers (92864, &c.) in Exercise 2, are to be placed; and the sum of all the eight numbers is 227898; and the like is to be understood regarding the others. In what is here given, the six exercises have been combined by twos and threes; and, should the teacher think it right to give exercises of such length, they might be combined by fours and fives, and even all the six might be taken.

It may be remarked, that each of the first fifteen of the foregoing combinations will give in reality *two* exercises. Thus, in the first, if the numbers in Exercise 1 be put below those in Exercise 2, there will be an entirely new succession of figures; and this new arrangement, especially after the lapse of a day or two, will afford the pupil a completely new exercise. For a like reason, each of the remaining twenty exercises will give, by transposition, no fewer than six distinct questions.

Additional exercises in subtraction will be scarcely necessary, as ample practice in its operations is afforded in performing exercises in division. Should more be desired, however, they may be formed at the moment, and their examination will present no difficulty.

In multiplication and division, great advantage may be obtained by employing methods of forming exercises depending on the 'casting out of the nines,' as explained in the Arithmetic, page 28. In the annexed example, the multiplicand is so assumed, that when the nines are cast out, there is no excess. Then it is plain that, in the answer, and in the several partial products, there should be no excess. Hence, when an exercise of this kind has been set or dictated to the pupil, and when he presents the work of it for examination, the teacher should mentally cast the nines out of the answer; and if there be no excess, it may be presumed that the work is right; but, as a check, he may test similarly one or more of the partial products. Should there be an excess, however, in the answer, let him try a partial product about the middle of the work; if this be right, the error must be *after* it; otherwise, it must be *before* it. He then tries another partial product, about the middle of the part in which the error lies; and thus, in a few seconds, he is enabled to draw his pen or pencil through the line containing the error, and return the work to the pupil for correction. In the annexed example, the answer is wrong, giving an excess of 3. Trying then the second partial product, we find it to be correct, as there is no excess; and, as the third gives an excess of 3, it and the addition must be rectified.

Exam. 1.
6278643
5824
25114572
12557286
50259144
31393215
36569816832

It is desirable to vary the mode of forming the exercises, lest —which will be found to be extremely rare—some of the smarter pupils should discover the plan. In the annexed example, the unit figure of the multiplicand is made 1, and there is no excess in the figures before it. Then the excess in the answer must be the same as in the multiplier; and, in each of the partial products wanting the last figure, there should be no excess. In the present case, the excess in the multiplier is 5, while in the answer it is only 4. Then, trying the second product, we find it right; as also the third. The fourth, wanting the last figure, gives an excess of 8, and is therefore wrong.

Exam. 2.
57871
8537
405097
173613
289355
463768
494844727

In Example 3, the last figure of the multiplicand is also 1, and the excess of the figures before it is 2 less than 9. In this case, the excess of the multiplier and the answer, taken as if they formed a single number, should be nothing; and the same should be the case in each of the partial products, with its last figure taken twice. Here, the excess in the multiplier and the product, taken jointly, is 1. Then, as the partial products are found to be correct, the error must be in the addition.

Exam. 3.

7451
8462
14902
44706
29804
59608
63060362

It is plain, that the property of the number 11, mentioned in the note in the Arithmetic, page 29, might be employed in a similar way, and, in making use of that property, the principle of the method would be very effectually disguised. The foregoing method, however, is simpler and easier in practice.

The same principles may be extended to division, with still more advantage. In the annexed example, the divisor is assumed so as to have no excess; and, the divisor consisting of *three* figures, the *four* figures, 7524, after the first of the dividend, are also taken so as to give 0 for excess. The remaining figures of the dividend, in all such exercises, are to be ciphers or nines, or a mixture of both. Then, it is plain that in each of the successive remainders, with the following figures annexed (1864, 2989, &c.), the excess ought to be 1, the first figure of the dividend; while in each of the products (1566, 2449, &c.), there ought to be no excess. Now, by glancing at the last remainder, 531, we see that the excess is 0, while it ought to be 1. To find the place of the error, we try the excess of 7020 (near the middle), and as this is also wrong, the error must be farther back. Going back four lines, therefore, we find that 2989 gives the true excess, while the next product, 2449, gives an excess of 1, and is therefore wrong.

Exam. 4.

783) 1752490099 (2236896
1566
1864
1566
2989
2449
5400
4698
7020
6264
7569
7047
5229
4698
531

If the divisor consisted of *four* figures, then the *five* figures after the first of the dividend should give no excess; and so in other cases.

If the pupil study Arithmetic for an hour or upwards at once, the teacher will find it to be convenient, and to save time, if, at the commencement of the period, he set an exercise such as the one here given, and having the dividend beginning with 1; then, for a second exercise, change that 1 into 2; for a third, the 2 into 3, &c., retaining the divisor, and all the rest of the dividend, the same throughout. By this means, he will be saved the time and trouble of forming new questions, and he will see the

number of exercises done by the pupil. For the sake of variety in such cases, the divisor may be occasionally changed.

In all cases also the pupil should be required to prove his work by multiplication; as by this means, every exercise so performed will give him practice, not only in division, but also in addition, subtraction, and multiplication; and by being so employed for some time, he will rapidly acquire readiness and accuracy in the performance of operations, and will have ease and pleasure in the farther prosecution of the study of arithmetic. If this method of proof be followed, the learner may be removed pretty early from multiplication to division; as, in proving his exercises in division, he will have sufficient practice in multiplication.

The exercises in reduction, given in the Arithmetic, may be doubled in number, if, a short time after the pupil has performed them, the teacher make the answers given in the book serve as new questions. Thus, from Exercises 1 and 2, the following questions may be formed: (1.) Reduce 81844 pence to pounds; (2.) Reduce 46975 halfpence to pounds; and the answers will be £341 - 0 - 4, and £97 - 17 - $3\frac{1}{2}$.

In like manner, the answers of the exercises in compound multiplication will serve as questions in compound division, while those in division will afford others in multiplication. Thus, from the 13th in multiplication, we get £26 - 9 - 1 + 14; and the answer will be £1 - 17 - $9\frac{1}{2}$; while from the 15th in division, we get £0 - 16 - $6\frac{1}{4}$ × 49: and the answer will be £40 - 9 - $6\frac{1}{4}$ (= £40 - 10 - 0 − $5\frac{3}{4}$*d.*).

In proportion the exercises may be varied by varying the order in which the terms of the ratio-equation are taken. Thus, from Exercise 17, page 103, the following exercises can be made: (1.) If 171 cwt. of sugar cost £216, how much could be bought for £120 at the same rate?" (2.) If 95 cwt. cost £120, how much could be bought for £216 at the same rate? (3.) If 95 cwt. cost £120, what would 171 cwt. cost at the same rate? Any exercise in the book, also, may be converted into another having the same answer, by either multiplying or dividing the terms that are to be placed second and third, by any two numbers; and, at the same time, multiplying or dividing the first term by the same. Thus, from Exercise 23, by multiplying 22 by 4, and each of the other two terms by 2, we get the following question: If 88 yards of ribbon cost 62*s.*, what will 30 yards cost? From Exercise 22, also, by dividing £54 by 6, and the two remaining terms first by 2 and 3, and then by 3 and 2 respectively, we get the two following questions: (1.) If 189 yards of linen cost £9, how much might be bought for £7 - 16 - 10 at the same rate? (2.) If 126 yards of linen cost £9, how much might be bought for £11 - 15 - 3 at the same rate?

The exercises in practice will furnish others having the same answers, if the quantity be multiplied, and the price divided by the same number; or if the quantity be divided by any number, and the price be multiplied by the same. Thus, from Exercise

13, by dividing and multiplying by 3, we get 563 at 14*s.* 7½*d.*; while, by multiplying and dividing by 2, Exercise 36 gives 498 at £2 - 16 - 10½. In like manner, by multiplying and dividing by 2 in Exercise 75, we get 234 cwt. 2 qrs. 14 lbs. at 8*s.* 8*d.*

In interest, a question having the same answer may be formed from any of the exercises by similar means, such as by doubling the principal and halving the time, &c.; and in discount, wrought in the common way, the debt and the time may be treated similarly. It is easy to see how this principle may be applied to the exercises in compound multiplication and division.

Numerous exercises in the contracted method of multiplying and dividing decimals, will be obtained from the table in the Arithmetic, page 341. Thus, if the product of the numbers in any one of the four columns opposite to 15 years and 19 years be taken, the answer, differing slightly at the end, will be found in the same column opposite to 34 years, the sum of 15 and 19: and if the number in one column opposite to 46 years be divided by the one in the same column opposite to 29 years, the answer will be found in the same column opposite to 17 years, the difference of 46 and 29. Exercises of a similar kind might also be found from those in involution and evolution.

In the extraction of roots, the exercises in the book will readily afford others, the answers of which will be easily found. Thus, from Exercise 1 in the square root, by multiplying the given number, 5, successively by 4, 9, 16, 25, &c., we get 20, 45, 80, 125, &c.; the roots of which are respectively twice, three times, four times, five times, &c., 2·236068, the answer in the book; while, if 5 were divided successively by the same numbers, we should have 1¼, ·5, ·3125, ·2, &c.; the roots of which would be respectively the half, third, fourth, fifth, &c., of 2·236068. In the cube root, if any of the given numbers be multiplied by 8, the root of the product will be double of that in the book, but if it be divided by 8, the answer will be only one half of the same.

Such are a few expedients, out of many, that may be employed by arithmetical teachers in diminishing labour and saving time, and many others will readily suggest themselves, both in reference to the rules here adverted to, and to others.

It may be remarked, also, that, to persons learning proportion, practice, and interest, the exercises in the article on mental arithmetic may be given, with advantage, to be wrought in the common manner; and that many of the exercises in practice will serve well as questions in proportion; and several in proportion, as exercises in practice.

As to the method of teaching arithmetic, it cannot be too strongly impressed on the consideration of the teacher, that, when it is at all practicable, the pupil should never be required to perform an operation without being made acquainted with the principles on which it depends. It is thought by many, that the reasons of the various rules in arithmetic are too difficult for the younger pupils, and that time would be lost in attempting to make such pupils comprehend them. The very reverse of this

opinion, however, is indicated by intelligent consideration, and is strongly confirmed by ample experience. Young pupils will understand simple and familiar explanations of the principles of arithmetic much better than is generally believed: and whatever time and pains may be employed in giving such explanations will be amply repaid by the training and strengthening of the mind, so produced. Pupils thus instructed in the elementary parts of arithmetic, will afterwards learn the more advanced parts with far more ease and pleasure; and, what is of the greatest consequence, their reasoning powers will be called into action, and will be fitted for being applied with success in relation to other subjects. The teacher of any branch of education, besides communicating knowledge to his pupils, ought to cultivate their reasoning powers, and to accustom their minds to regular and useful exertion. When a teacher, however, without giving the reason of the process, merely tells the pupil to carry 1 for 10 in the fundamental rules; or, in multiplication of fractions, to find the product of the numerators and the product of the denominators for the numerator and denominator of the answer, he does communicate the requisite knowledge, but he altogether fails in giving proper culture to the mind, and in preparing it for overcoming the difficulties that must be perpetually presenting themselves in after life.

It is true, indeed, that as the education of a great number of pupils must be necessarily limited, they ought, as a matter of utility, to be instructed in the application of arithmetic to subjects, with the principles of which they are unacquainted. Though a person may not have studied the mathematical theory of the circle, there is no reason, that he should not be able to go through with computations regarding the area of that figure, the length of its circumference, &c.; or that he should not be taught to extract the cube root, though he may not be able to follow out the reasoning in the note at the end of the Arithmetic, on Horner's method of resolving equations. In such cases, however, particular care should be taken to impress on his mind, that he is performing processes which admit of perfectly satisfactory explanations; and that, as he is now unable to understand those explanations, he ought for the present to receive the rules as true, and to endeavour to prepare himself for understanding their demonstrations afterwards.

It will be found to be of no inconsiderable advantage to arithmetical pupils, and, in particular, to those of promising talents, to have their attention frequently directed by the teacher to the relations and nature of the numbers that occur in their calculations. This will be promoted by habituating them to employ several of the contracted processes pointed out in the Arithmetic, in Rule IX., page 122; in fractions, Rule II., p. 194; and in several other parts of the work. By such training the learner will become a much abler, and a much more expert and intelligent arithmetician than he otherwise would; and his mind will be well prepared for the study of mathematics, should circum-

stances or inclination cause him to enter on that fine field of inquiry. This latter object would also be promoted by inviting (not requiring) the pupils to attend to questions of an interesting kind, and of degrees of difficulty suited to their proficiency, such as a number of the miscellaneous questions in the Arithmetic, commencing in page 304.

In directing the arithmetical studies of learners, the teacher will find it beneficial to give those studies, to some extent, a practical bearing. As soon as the pupil has learned simple multiplication, he may be caused with advantage to work questions in mensuration, such as Exer. 4, Arithmetic, page 331; and other similar questions regarding parallelograms, triangles, trapeziums, parallelopipeds, &c., having their dimensions expressed in whole numbers. Such questions the teacher will feel no difficulty in forming; and he will find it useful to the learner, to encourage him to take the dimensions of fields, rooms, boxes, and other objects, and to go through with the computations for finding their contents. When the pupil has learned simple division, he may be required to work Exer. 25, Arithmetic, page 340, and others of a similar kind. After he has become acquainted with proportion, he will work, with interest and advantage (by the second and third parts of Rule VIII.), Exer. 20, Arithmetic, page 334, and other similar ones; such as to find the diameter of a circular tree, column, or cask, if its girt be found by measurement to be 110 inches. In such cases, the remainders may be expressed in vulgar fractions. When the learner has studied vulgar and decimal fractions, his attention may be directed to the remaining parts of the article on mensuration; and, in every case, he should be encouraged to make measurements of suitable objects around him, and to employ the dimensions so found, as data for computations.

Though it was obviously incompatible with the nature of the Appendix on mensuration, subjoined to the Arithmetic, to attempt the demonstration of the rules, yet the intelligent teacher may readily give, even to pupils unacquainted with mathematics, explanations and illustrations, which will be very useful, and which will prepare their minds for following out the strict demonstrations afforded by geometry. Assistance, with regard to this object, will be obtained from the corollaries to the forty-sixth proposition of the first book of the author's edition of Euclid [Thomson's Euclid]; and from the treatises on mensuration, particularly those by Hutton and Bonnycastle.

The practical application of arithmetic should not be confined to mensuration, but should be extended to various other subjects. In the different sections of the Arithmetic, many questions are given, having this object in view; and it will be easy for the teacher to form others. Such questions he should register for use in a book (perhaps with the operations at full length, to enable him to examine the work of the pupil with more facility); and in making them out, he would find it advantageous to form them of such a kind that a large proportion of them may be of

a local character. Thus, if his pupils belong to an agricultural district, he might be prepared to propose, from time to time, questions regarding the prices of grain and other agricultural produce, the rents of lands, and similar subjects; and teachers in manufacturing districts, those in towns, where the principal business is shopkeeping, and those in other towns, where shipping, in its various branches, forms the chief occupation of the people, besides those in many other localities, will all, by proper consideration, be able to form exercises that may be interesting and useful to the pupil, in addition to those of a more general character, given in the Arithmetic. In the prescribing of all such questions, care should be taken that they are suited to the comprehension of the learner; as none that are too difficult ought ever to be forced on his attention.

In conclusion, care should always be taken, that, in the study of arithmetic, as well as in every other part of education, the pupils should be made to feel that they are addressed and treated, not as mere passive machines, as has unfortunately been too generally done, but as thinking, rational beings, who are to work out their own improvement by the friendly aid of their teacher, and by deriving from him, and from the books which they read, the advantage of the experience and researches of the men of the present and of past times. By such intellectual training, the interests of the rising generation, and of society at large, will be greatly advanced; and the teacher, by acquiring character, will promote his own interest; and he will have the satisfaction of feeling that he has discharged his duty in a proper and conscientious manner.

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